REVIEW E – Beyond the AP

Summary of Topics for Chapter E:

Volumes of revolution by the shell method
Surface area on a volume of revolution
Integration using a trigonometric substitution
❖Epsilon-delta limit definitions
Proving a two-sided limit
Proving a limit at infinity
Proving an infinite limit
Proving a limit does not exist.

Suggested Review Problems from the Textbook

pg. 91-92 Review Exercises: 5, 7
pg. 205-208 Exercises: 102, 103
pg. 242-244 Review Exercises: 47 (prove the limit)
pg. 515-516 Review Exercises: 22abd, 25, 27, 37, 38
pg. 591-592 Review Exercises: 27, 28, 29

Additional Suggested Review Problems

1. Prove: $\lim_{x\to 4} f(x) \neq 13$ where $f(x) = \begin{cases} x^2 - 3 & x \le 4 \\ x + 9.2 & x > 4 \end{cases}$

2. Prove: $\lim_{x\to -\infty} \frac{x+1}{2x+1} = \frac{1}{2}$

3. Prove: $\lim_{x\to -3^-} \frac{x}{x+3} = +\infty$

Answers

Suggested Review Problems from the Textbook

Pg. 591-592 Review Exercises

22. $\frac{176\pi}{15}$

38. $\frac{152\pi}{3}$

Pg. 591-592 Review Exercises

28. $\sqrt{x^2-9} - 3 \bullet \arctan\left(\frac{\sqrt{x^2-9}}{3}\right) + C$

Additional Suggested Review Problems

1. Prove: $\lim_{x\to 4} f(x) \neq 13$ where $f(x)=\begin{cases} x^2-3 & x\le 4 \\ x+9.2 & x>4 \end{cases}$

   Choose $\varepsilon = 0.1$ and any $\delta > 0$.
   If $0 < x-4 < \delta$, then
   $0.2 < x-3.8 < \delta + 0.2$
   $0.2 < x+9.2-13 < \delta + 0.2$
   $0.2 < |x+9.2-13| < \delta + 0.2$
   $|x+9.2-13| > 0.2$
   $|x+9.2-13| > 0.2 > 0.1 = \varepsilon$
   Therefore $\lim_{x\to 4} f(x) \neq 13$

2. Prove: $\lim_{x\to -\infty} \frac{x+1}{2x+1} = \frac{1}{2}$

   Given $\varepsilon > 0$, choose $M = \frac{-1}{4\varepsilon} - \frac{1}{2}$

   If $x < M$, then $x < \frac{-1}{4\varepsilon} - \frac{1}{2} < 0$

   $2x < \frac{-1}{2\varepsilon} - 1 < 0$

   $2x+1 < \frac{-1}{2\varepsilon} < 0$

   $2(2x+1) < \frac{-1}{\varepsilon} < 0$

   $-2(2x+1) > \frac{1}{\varepsilon} > 0$

   $0 < \frac{1}{-2(2x+1)} < \varepsilon$

   $\left|\frac{1}{-2(2x+1)}\right| < \varepsilon$

   $\left|\frac{1}{2(2x+1)}\right| < \varepsilon$

   $\left|\frac{1}{2x+1} - \frac{1}{2}\right| < \varepsilon$

   Therefore $\lim_{x\to -\infty} \frac{x+1}{2x+1} = \frac{1}{2}$

   Preparation for the proof.

   $x < M \Rightarrow \left|\frac{x+1}{2x+1} - \frac{1}{2}\right| < \varepsilon$

   $\left|\frac{1}{4x+2}\right| < \varepsilon$

   $|4x+2| > \frac{1}{\varepsilon}$

   $4x < 4M$
   $4x+2 < 4M+2$
   $-4x-2 > -4M-2$

   $|4x+2| > -4M-2$ provided $x < \frac{-1}{2}$

   Let $-4M-2 = \frac{1}{\varepsilon}$

   $-4M = \frac{1}{\varepsilon} + 2$

   $M = \frac{-1}{4\varepsilon} - \frac{1}{2}$

   Note that since this makes $M < \frac{-1}{2}$ and

   therefore $x < \frac{-1}{2}$.

3. Prove: $\lim_{x \to -3^-} \frac{x}{x+3} = +\infty$

Given $N > 0$, choose $\delta = \min\left(\frac{3}{N}, 1\right)$

If $0 < -x - 3 < \delta$, then

$0 < -x - 3 < \frac{3}{N}$ and $0 < -x - 3 < 1$

$\frac{1}{-x-3} > \frac{N}{3} > 0$ $\quad$ $3 < -x < 4$

$-x > 3 > 0$

$\frac{-x}{-x-3} > \frac{N}{3} \bullet 3$

$\frac{x}{x+3} > N$

Therefore $\lim_{x \to -3^-} \frac{x}{x+3} = +\infty$

Preparation for the proof.

$0 < -x - 3 < \delta \Rightarrow \quad \frac{x}{x+3} > N$

$\frac{1}{-x-3} > \frac{1}{\delta} > 0$

Let $\delta < 1$

$0 < -x - 3 < 1$

$3 < -x < 4$

$-x > 3 > 0$

$\frac{-x}{-x-3} > \frac{3}{\delta} > 0$

$\frac{x}{x+3} > \frac{3}{\delta} > 0$

Let $N = \frac{3}{\delta}$

$\delta = \frac{3}{N}$