REVIEW 9 – Infinite Series

Summary of Topics for Chapter 9:

Sequences – convergent, bounded, monotonic
Geometric series and ❖telescoping series
❖Nth term test for divergence
❖*P*-series
❖Integral Test
❖Direct Comparison Test
Order of magnitude and ❖Limit Comparison Test
❖Alternating series – convergence and error bound
Absolutely versus conditionally convergent series
❖Ratio Test
❖Taylor polynomials and ❖LaGrange Error Bound
Power series – radius of convergence, interval of convergence, properties
❖Power series for elementary functions
❖Taylor series
TI-CAS:

- Graph a sequence.
- Write a Taylor polynomial for a given function.
- Evaluate a Taylor polynomial at a specific point.

Testing for Convergence: To test for convergence think through these steps in order:

Determine whether or not the limit of the n^{th} term is zero.
Determine if the series is a special type: p-series, geometric series, telescoping series, or alternating series.
Determine if the Ratio Test or Integral Test can be applied successfully.
Determine if a Direct or Limit Comparison test can be applied successfully.

Summing a Series: The only series for which you can actually find a sum:

Geometric series
Telescoping series
Recognizable Taylor series

Error Bounds: The only series for which you can bound the error of the sum:

Alternating series
Taylor series – Lagrange error bound

Suggested Review Problems from the Textbook

pg. 690-694 Review Exercises: 1, 9, 11, 15, 25, 26, 28, , 29, 31, 33, 39, 42, 43, 45, 47, 49, 51, 53, 55, 57,59, 60, 65, 68, 70, 71a, 73, 75, 77, 81, 82, 83, 85, 87, 97, 99, 105, 106, 108

Additional Suggested Review Problem

1. Given the function $f(x)$ whose fourth degree Taylor polynomial expansion centered at $x = 0$ is $P(x) = 2 + \frac{4}{5}x^2 + 5x^3 - x^4$.
   a. Find $f^{(3)}(0)$.
   b. Does $f(x)$ have a critical point at $x = 0$? If so, is it a relative maximum or a relative minimum?
   c. Does $f(x)$ have an inflection point at $x = 0$?

2. How many terms of the series $\sum_{n=1}^{\infty} (-1)^n \frac{3}{n^3}$ would have to be summed to have a sum correct to the nearest thousandth?

Answers

Suggested Review Problems from the Textbook

Pg. 690-694 Review Exercises

26. 8
28. 1
42. Converges because it is the difference of two convergent series, the first a convergent *p*-series and the second a convergent geometric series.
60. Converges by Ratio Test.
68. All derivatives of $y = \cos x$ involve the sine or cosine function both of whose maximum value is 1. Therefore we can choose $M = 1$. By trial and error on the number of terms we find that $|R_5| = \frac{1}{6!}(0.75 - 0)^6 = 0.000247 < 0.001$. Therefore a fifth degree Taylor polynomial is required. (However, remember that the fifth degree term is zero for the cosine expansion.) $\cos(0.75) \approx 1 - \frac{(0.75)^2}{2!} + \frac{(0.75)^4}{4!} = 0.7319$;
70. All derivatives of $y = e^x$ are $y = e^x$ which is an increasing function. So on the interval $(-0.25, 0)$ the maximum value would be at $x = 0$. Therefore $M = e^0 = 1$. By trial and error on the number of terms we find that $|R_3| = \frac{1}{4!}(-0.25 - 0)^4 = 0.000163 < 0.001$. Therefore a third degree Taylor polynomial is required. $e^{-0.25} \approx 1 + (-0.25) + \frac{(-0.25)^2}{2!} + \frac{(-0.25)^3}{3!} = 0.7786$
82. Start with the power series for $\frac{1}{1-x}$ Multiply all terms by $\frac{3}{2}$ to obtain an expansion for $\frac{3}{2-2x}$. Now substitute $\frac{-x}{2}$ in place of x to obtain $\frac{3}{2+x} = \frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8} - \frac{3x^3}{16} + \ldots \frac{(-1)^n 3x^n}{2^{n+1}} + \ldots$.

106. $\frac{x}{2}-\frac{x^2}{2\bullet 2\bullet 2!}+\frac{x^3}{2\bullet 3\bullet 3!}-\frac{x^4}{2\bullet 4\bullet 4!}+...+\frac{x^n}{2\bullet n\bullet n!}+...$

108. $x+\frac{x^2}{2\bullet 2!}+\frac{x^3}{3\bullet 3!}+\frac{x^4}{4\bullet 4!}+...\frac{x^n}{n\bullet n!}...$

Additional Suggested Review Problems

1. a. $5=\frac{f'''(0)}{3!}$. Therefore $f'''(0)=30$

   b. Since the term with x to the first power is missing, its coefficient must be zero. Therefore $f'(0)=0$, making $x=0$ the location of a critical point. Since $\frac{4}{5}=\frac{f''(0)}{2}$, making $f''(0)=\frac{8}{5}>0$, then $x=0$ is the location of a relative minimum.

   c. From the previous part we see that $f''(0)=\frac{8}{5}\neq 0$, hence there is no inflection point at $x=0$.

2. Notice that the series is alternating. By trial and error we find that the 15th term is $\frac{3}{15^3}=0.000889<0.001$. By the error bound theorem for summing an alternating series, the sum of 14 terms should be accurate to the nearest thousandth.