REVIEW 10 – Parametric, Vector, and Polar Equations

Summary of Topics for Sections 10.2 to 10.6:

Parametric curves
Vectors
Velocity, acceleration, and speed on a parametric curve
❖Length of a path
Tangent lines to a parametric or vector curve
❖First and Second derivatives of a parametric function
Polar curves, including conics
Derivative of a polar curve
❖Area inside a polar curve
Tangents to a polar curve at the origin
TI-CAS: Graph a pair of parametric equations or a polar equation.

Suggested Review Problems from the Textbook

pg. 758-760 Review Exercises: 25, 27, 29, 31, 33, 38, 39, 43, 48, 51, 53, 55, 69, 72, 83, 87, 90, 97, 101, 103, 106, 111, 117, 119

Additional Suggested Review Problems

1. Find all asymptotes of the graph of the parametric equations $x = \frac{1}{t}$ and $y = \frac{t}{t+1}$.

2. A particle travels along a curve with velocity vector $\langle \sin t, 1 - \cos t \rangle$ for $t \geq 0$. At time $t = 0$ the particle is at the origin.

   a. Find the distance traveled by the particle from $t = 0$ to $t = \frac{2\pi}{3}$.

   b. Find the acceleration vector at time $t = \pi$.

   c. Find the first time at which the particle is moving parallel to the *y*-axis. Is it traveling upward or downward at that moment?

   d. Where is the particle at time $t = 1$?

   e. Find the first time after $t = 0$ that the particle is at rest.

3. A projectile is launched over level ground from a height of 2 meters at a launch angle of 60°. Its initial speed is 500 m/sec.

   a. Write an acceleration vector and a velocity vector for its motion at any time *t*.

   b. Write a pair of parametric equations for its position at any time *t*.

   c. To the nearest whole meter, how high overhead will the projectile be when it is 2 km downrange?

   d. To the nearest whole meter, what is the highest the projectile will go?

Answers

Suggested Review Problems from the Textbook

Pg. 591-592 Review Exercises

38. $y = 4\sin\theta\cos\theta = 4\sin^2\theta \bullet \frac{\cos\theta}{\sin\theta} = 4\sin\theta\cot\theta = 4\sin^2\theta \bullet \frac{x}{2}$. Now since $x = 2\cot\theta$, then

$x^2 = 4\cot^2\theta = 4(\csc^2\theta - 1) = 4\csc^2\theta - 4$. Solving this equation for $\csc^2\theta$ yields

$\frac{x^2+4}{4} = \csc^2\theta$ or inverting to get $\frac{4}{x^2+4} = \sin^2\theta$. Substituting this into the first equation for y

yields $y = 4 \bullet \frac{4}{x^2+4} \bullet \frac{x}{2} = \frac{8x}{x^2+4}$.

48. a. $\frac{dy}{dx} = \frac{-e^{-t}}{e^t} = \frac{-1}{e^t}$. There are no horizontal tangents; b. $y = \frac{1}{x}$; c.

72. $3x^2 - 2x + 4y^2 - 1 = 0$

90. check on your TI-CAS

106. $2\left[\frac{1}{2}\int_0^{\pi/3} 2^2\,d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} (4\cos\theta)^2\,d\theta\right] = 4.913$

Additional Suggested Review Problems

1. $y = 0$ and $x = -1$
2. a. 2
   b. $\langle -1, 0\rangle$
   c. $t = \pi$. It is moving upward.
   d. $x(t) = \int_0^t \sin u\,du + x(0) = 0.460$; $y(t) = \int_0^t (1-\cos u)\,du + y(0) = 0159$
   e. $t = 2\pi$
3. a. $a(t) = \langle 0, -9.8\rangle$; $v(t) = \langle 250, -9.8t + 250\sqrt{3}\rangle$
   b. $x(t) = 250t$ and $y(t) = -4.9t^2 + 250t\sqrt{3} + 2$
   c. 3153 meters
   d. 9568 meters