# PROBLEM SHEET S1 – Semester I Review

**Questions marked with a ♦ allow use of a calculator. Questions 1 – 22 are multiple choice.**

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1. At $x = 3$ the function given by $f(x) = \begin{cases} x^2 & x < 3 \\ 6x - 9 & x \geq 3 \end{cases}$ is

   A. undefined
   B. continuous but not differentiable
   C. differentiable but not continuous
   D. neither continuous nor differentiable
   E. both continuous and differentiable

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2. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$

   A. $-6$  B. $-\frac{9}{2}$  C. $-\frac{7}{2}$  D. $-3$  E. $-\frac{1}{2}$

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3. If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

   A. $\frac{3}{2}$  B. $\frac{1}{2}$  C. $0$  D. $\frac{-3}{2}$  E. nonexistant

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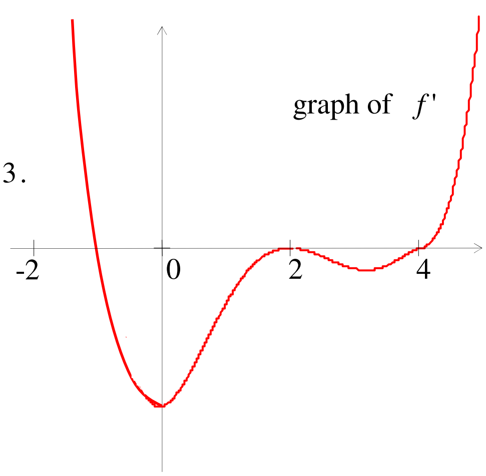
4. Let $f$ be a function that has domain $[-2, 5]$. The graph of $f'$ is shown at right. Which of the following statements are TRUE?

   I. $f$ has a relative maximum at $x = -1$.
   II. $f$ has an absolute minimum at $x = 0$.
   III. $f$ is concave down for $-2 < x < 0$.
   IV. $f$ has inflection points at $x = 0$, $x = 2$, and $x = 3$.




   A. I, II, IV only
   B. I, III, IV only
   C. II, III, IV only
   D. I, II, III only
   E. I, II, III, IV

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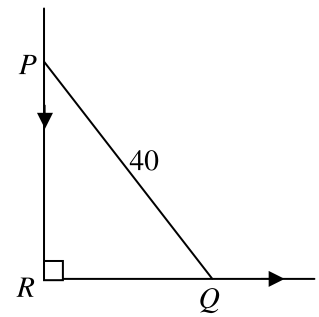
5. In the figure at right, $PQ$ represents a 40-foot ladder with end $P$ against a vertical wall and end $Q$ on level ground. If the ladder is slipping down the wall, what is the distance $RQ$ at the instant when $Q$ is moving along the ground $\frac{3}{4}$ as fast as $P$ is moving down the wall?




A. $\frac{6}{5}\sqrt{10}$ B. $\frac{8}{5}\sqrt{10}$ C. $\frac{80}{\sqrt{7}}$

D. 24 E. 32

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6. ♦Let $g$ be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, $g$ is decreasing most rapidly when $t =$

A. 0.949 B. 2.017 C. 3.106 D. 5.965 E. 8.000

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7. Let $f$ be a polynomial function with degree greater than 2. If $a \ne b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of $x$ between $a$ and $b$?

I. $f(x) = 0$

II. $f'(x) = 0$

III. $f''(x) = 0$

A. None B. I only C. II only D. I and II only E. I, II, and III

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8. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

A. $\frac{1}{\pi}$ B. $\frac{1}{2}$ C. $\frac{2}{\pi}$ D. 1 E. 2

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9. For small values of $h$, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

A. $4 + \frac{h}{32}$ B. $2 + \frac{h}{32}$ C. $\frac{h}{32}$ D. $4 - \frac{h}{32}$ E. $2 - \frac{h}{32}$

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10. The acceleration of a particle at time $t$ moving along the $x$-axis is given by $a(t) = 6t$. At the instant when $t = 0$, the particle is at the point $x = 2$ moving with velocity $v = -2$. The position of the particle at $t = \frac{1}{2}$ is

A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$ E. $\frac{9}{8}$

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11. The function $g(x) = \int_0^x \sin t\,dt$ has an inflection point at $x =$

A. 0 B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{2\pi}{3}$ E. $\pi$

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12. ♦An approximation for $\int_{-1}^{2} e^{1.5x}dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

A. 25.6 B. 25.7 C. 25.8 D. 25.9 E. 30

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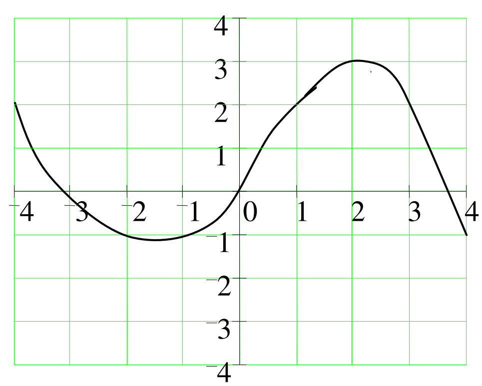
13. The graph of a function $f$ whose domain is the interval $[-4, 4]$ is shown in the figure. Which of the following statements are true?




I. The midpoint approximation of the area under the curve from $x = 0$ to $x = 4$ using only two subdivisions is 8.

II. The left sum approximation of the total area between the curve and the $x$-axis from $x = -1$ to $x = 3$, using 4 equal subdivisions, is 6.

III. The left-sum approximation of $\int_{-1}^{3} f(t)dt$ with 4 equal subdivisions is 6.

A. I only B. II only C. II and III only D. I and II only E. I, II, and III

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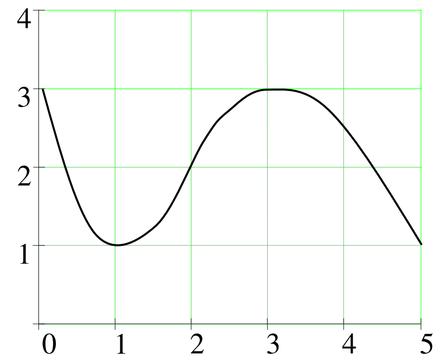
14. Use the Trapezoidal Rule with $n = 4$ to approximate the integral $\int_1^5 f(x)dx$ for the function $f$ whose graph is shown at the right.




A. 8 B. 8.5 C. 9

D. 9.5 E. 10

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15. If $f(x)$ is a linear function with slope $m$, then $\int_a^b f'(x)dx$ has the value

A. 0 B. $b - a$ C. $m(b - a)$ D. $m(b^2 - a^2)$ E. $\frac{m}{2}(b^2 - a^2)$

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16. ♦The approximate average rate of change of the function $f(x) = \int_0^x \sqrt{1 + \cos^2(t)}dt$ over the interval $[1, 5]$ is nearest to

A. 1.09 B. 1.12 C. 1.15 D. 1.18 E. 1.21

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17. $\int x\sqrt{3 + x}dx =$

A. $\frac{2}{5}(3+x)^5 - 2(3+x)^3 + C$

B. $\frac{2}{5}(3+x)^{5/2} - 2(3+x)^{3/2} + C$

C. $\frac{1}{2}(3+x)^2 - \frac{3}{2}(3+x) + C$

D. $\frac{1}{2}(3+x)^4 - \frac{3}{2}(3+x)^2 + C$

E. $\frac{1}{2}x^4 - \frac{3}{2}x^2 + C$

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18. If $\frac{dy}{dx} = x^2 y$, then $y$ could be

A. $3\ln\left(\frac{x}{3}\right)$ B. $e^{x^3/3} + 7$ C. $2e^{x^{3/3}}$ D. $3e^{2x}$ E. $\frac{x^3}{3} + 1$

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19. ♦During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1000 people are infected when the epidemic is first discovered, and 1200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

A. 343 B. 1343 C. 1367 D. 1400 E. 2057

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20. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \left(\frac{P}{5000}\right)\right)$ where the initial population $P(0) = 3{,}000$ and $t$ is the time in years. What is $\lim_{t \to \infty} P(t)$?

A. 2,500 B. 3,000 C. 4,200 D. 5,000 E. 10,000

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21. $\int_2^3 \frac{3}{(x-1)(x+2)} dx$ is

A. $-\frac{33}{20}$ B. $\frac{-9}{20}$ C. $\ln\left(\frac{5}{2}\right)$ D. $\ln\left(\frac{8}{5}\right)$ E. $\ln\left(\frac{2}{5}\right)$

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22. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours in how many hours will the number of bacteria triple?

A. $\frac{3\ln 3}{\ln 2}$ B. $\frac{2\ln 3}{\ln 2}$ C. $\frac{\ln 3}{\ln 2}$ D. $\ln\left(\frac{27}{2}\right)$ E. $\ln\left(\frac{9}{2}\right)$

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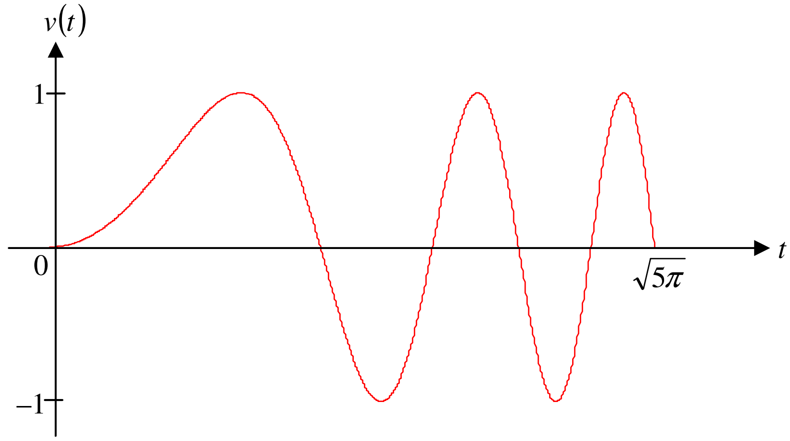



23. ♦ A particle moves along the $x$-axis so that its velocity $v$ at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of $v$ is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time $t$ is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

a. Find the acceleration of the particle at time $t = 3$.

b. Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

c. Find the position of the particle at time $t = 3$.

d. For $0 \leq t \leq \sqrt{5\pi}$, find the time $t$ at which the particle is farthest to the right. Explain your answer.

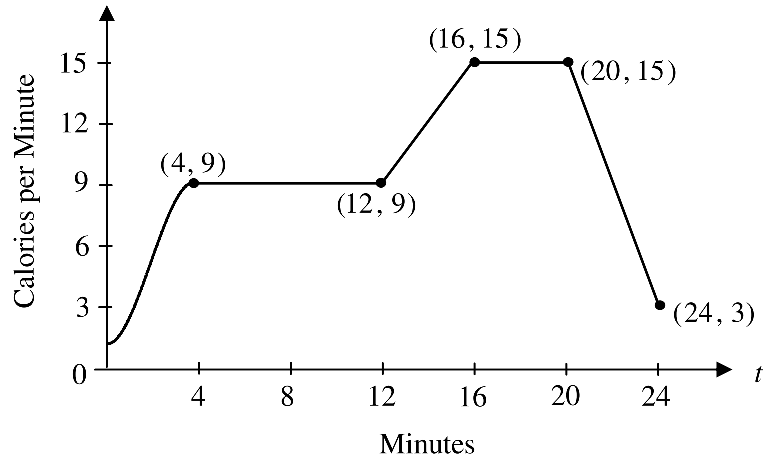
24. Let $f$ be the function given by $f(x) = \sqrt{x-3}$.

a. Sketch a graph of $f$ and shade the region $R$ enclosed by the graph of $f$, the $x$-axis, and the vertical line $x = 6$.

b. Find the area of the region $R$ described in part a.

c. Rather than using the line $x = 6$ as in part a., consider the line $x = w$, where $w$ can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of $f$, the $x$-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

d. Let $A(w)$ be as described in part c. Find the rate of change of $A$ with respect to $w$ when $w = 6$.



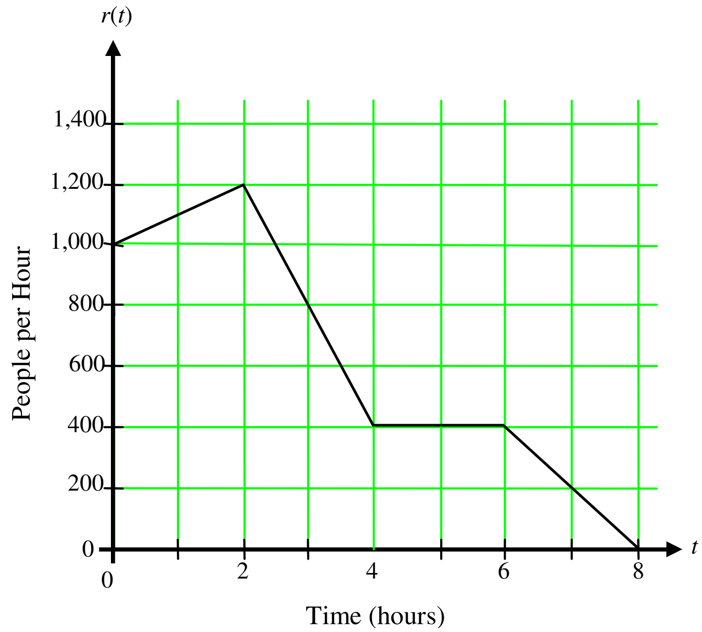

25. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function *f*. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and $f$ is piecewise linear for $4 \le t \le 24$..

a. Find $f'(22)$. Indicate the units of measure.

b. For the time interval $0 \le t \le 24$, at what time $t$ is $f$ increasing at its greatest rate? Show the reasoning that supports your answer.

c. Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

d. The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find $c$ so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.




26. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time $t$ is measured in hours from the time the ride begins operation.

    a. How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

    b. Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

    c. At what time $t$ is the line for the ride the longest? How many people are in line at that time? Justify your answers.

    d. Write, but do not solve, an equation involving an integral expression of $r$ whose solution gives the earliest time $t$ at which there is no longer a line for the ride.

# Answer

1. E
2. B
3. E
4. B
5. E
6. B
7. C
8. D
9. B
10. E
11. C
12. A
13. D
14. B
15. C
16. D
17. B
18. C
19. C
20. E
21. D
22. A

23. a. $a(t) = 2t\cos(t^2)$; $a(3) = -5.467$
 b. 1.702
 c. 5.774
 d. $\sqrt{\pi}$

24. a.
 b. $\int_3^6 \sqrt{x-3}\,dx = 2\sqrt{3}$.
 c. $\int_3^w \sqrt{x-3}\,dx$
 d. $A'(w) = \sqrt{w-3}$ ; $A'(6) = \sqrt{3}$

25. a. $-3$ calories per minute squared
 b. $t = 2$
 c. 132 calories
 d. $c = 4$

26. a. $\int_0^3 r(t)dt = 16(200) = 3200$
 b. $r(t) > 800$ when $2 < t < 3$. Since people get on the ride at a rate of 800 people per hour, the line is increasing between $t = 2$ and $t = 3$.
 c. $F(x) = \int_0^x r(t)dt - 800x$ represents the number of people in line at time $t = x$.
 $F'(x) = r(x) - 800$
 $F'(x) = 0$ when $r(x) = 800$, which occurs at $x = 3$. Therefore $x = 3$ is the location of a critical point. Since $F'(x) > 0$ on $(0, 3)$ and $F'(x) < 0$ on $(3, 8)$, $x = 3$ is the location of an absolute maximum.
 d. $800x = \int_0^x r(t)dt$ or

 $0 = \int_0^x [r(t) - 800]dt$