

PROBLEM SHEET 8 – Integration Techniques

Some problems on this sheet are adapted from *Calculus; Concepts and Applications*, by Paul Foerster. California, Key Curriculum Press, 1998.

1. Infinite Paint Bucket Paradox: Consider the graph of $f(x) = \frac{-1}{x}$.
 - a. Make an informal sketch of the graph of $f(x)$ from $y = -1$ to $y = -\infty$.
 - b. TI-CAS allowed: Find the length of the curve that you sketched.
 - c. NO TI-CAS: Find the area of the region R that lies between $f(x)$ and the y -axis from $y = -1$ to $y = -\infty$.
 - d. TI-CAS allowed: Now rotate the region R about the y -axis. The integral $\int_0^1 2\pi x \sqrt{1 + [f'(x)]^2}$ represents the lateral surface area of the resulting solid. Find the lateral surface area.
 - e. NO TI-CAS: Imagine the solid to be an infinitely deep paint bucket filled with paint. Find the volume of paint this bucket could hold.
 - f. Contrast your answers to parts d. and e. Why is the problem called the Paint Bucket Paradox?

2. NO TI-CAS: Make an informal sketch of the area represented by the integral $\int_0^3 \lceil x \rceil dx$. Then use simple geometry to evaluate the integral.

3. Consider the function $f(x) = 2^x - \frac{|x-2|}{x-2}$ from $x = 1$ to $x = 3$.
 - a. Use your TI-CAS to make a detailed sketch of the curve.
 - b. Write a piecewise function that is equivalent to $f(x)$ without using an absolute value expression.
 - c. NO TI-CAS: Find the area between the curve and the x -axis.

Answers

1. a.
b. ∞
c.
d. ∞
e. π
- 2.
3. c. 8.656