PROBLEM SHEET 7 – Applications of Integration

Some problems on this sheet are taken from *Calculus; Single Variable*, second edition by Hughes-Hallett, Gleason, et al. New York, John Wiley & Sons, 1998.

You should expect to use your TI-CAS to evaluate most of the integrals on this problem set.

1. A rod has length 2 meters. At a distance x meters from its left end, the density of the rod is given by $p(x) = 2 + 6x$ g/m.
   a. Write a Riemann sum approximating the total mass of the rod.
   b. Find the exact mass by converting the sum into an integral.

2. The density of cars (in cars per mile) down a 20-mile stretch of the Pennsylvania Turnpike can be approximated by $p(x) = 300\left(2 + \sin\left(4\sqrt{x + 0.15}\right)\right)$, where x is the distance in miles from the Breezewood toll plaza.
   a. Sketch a graph of this function for $0 \le x \le 20$. (Use your TI-CAS.)
   b. Write a sum that approximates the total number of cars on this 20-mile stretch.
   c. Find the total number of cars on the 20-mile stretch.

3. Suppose you want to find the total mass of a 3×5 rectangular sheet, whose density per unit area at a distance x from one of the sides of length 5 is $\dfrac{1}{1+x^4}$.
   a. Write a Riemann sum that approximates the total mass.
   b. Find the total mass.

4. The density of oil in a circular oil slick on the surface of the ocean at a distance r meters from the center of the slick is given by $p(r) = \dfrac{50}{1+r}$ kg/m^2.
   a. If the slick extends from $r = 0$ to $r = 10{,}000$ meters, find a Riemann sum approximating the total mass of oil in the slick.
   b. Find the exact value for the mass of oil in the slick by turning your sum into an integral and evaluating it.
   c. Within what distance r is half the oil of the slick contained?

5. The soot produced by a garbage incinerator spreads out in a circular pattern. The depth, $H(r)$, in millimeters, of the soot deposited each month at a distance r kilometers from the incinerator is given by $H(r) = 0.115e^{-2r}$.
   a. Write a definite integral giving the total volume of soot (in cubic meters) deposited within 5 kilometers of the incinerator each month.
   b. Evaluate the integral.

6. A rod of length 3 meters with density $p(x) = 1 + x^2$ g/m is positioned along the positive *x*-axis, with its left end at the origin. Find the total mass of the rod.

7. Water is flowing in a cylindrical pipe of radius 1 inch. Because water is viscous and sticks to the pipe, the rate of flow varies with the distance from the center. The speed of the water at a distance r inches from the center is $10\left(1 - r^2\right)$ inches per second. What is the rate (in cubic inches per second) at which water is flowing through the pipe?

8. Given a region bounded by $y = \ln x$, $y = 0$, $x = 0$, and $y = -1$. Find the volume of the solid formed when the region is revolved about the *x*-axis.

Answers

1. a. $\sum_{i=1}^{n} (2 + 6x_i)\Delta x_i$

   b. $\int_0^2 (2 + 6x)dx = 16$ grams

2. b.

   c. 11513 cars

3. a. $\sum_{i=1}^{n} \frac{5}{1 + x_i^4}\Delta x_i$

   b. 5.5

4. a. $\sum_{i=1}^{n} \frac{2\pi r(50)}{1 + r_i}\Delta r_i$

   b.

   c. 5003.643 meters

5. a. $V = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} 2\pi(1000 r_i) \bullet \left(\frac{0.115 e^{-2r_i}}{1000}\right)(1000\Delta r_i)$

   $V = \int_0^5 2\pi(1000r)\left(\frac{0.115 e^{-2r}}{1000}\right)(1000)dr$

   b. 181 cubic meters

6. 12 g

7. $\int_0^1 2\pi r(10)\left(1 - r^2\right)dr = 15.708$ cubic inches per second.

8. 1.660