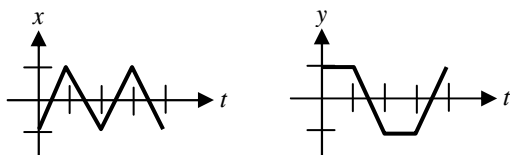


PROBLEM SHEET 10 – Parametric, Vector, and Polar Equations

Some problems on this sheet are taken from *Calculus; Single Variable*, second edition by Hughes-Hallett, Gleason, et al. New York, John Wiley & Sons, 1998.

- An object travels such that its x and y coordinates are given by $x = 3 - 2t$ and $y = t$. Find when and where it crosses the line $y = 2x + 5$.
- Graph the Cartesian curve described parametrically by the graphs below. Mark the starting point of the curve and use arrows to indicate direction of motion.



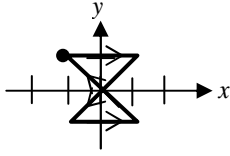
- Find the component form of the vector \vec{v} with head at $(5, 3)$ and tail at $(1, 1)$.
- Find the head of a vector \vec{v} with components $\langle 10, -6 \rangle$ and tail at $(13, 7)$.
- Find the tail of a vector \vec{v} with components $\langle -3, 8 \rangle$ and head at $(0, 4)$.
- Find vector \vec{v} in component form if $\vec{w} = \langle -8, 5 \rangle$ and $\vec{u} = \langle -3, 6 \rangle$, and $\vec{v} = 3\vec{u} - 2\vec{w}$.
- If $\vec{v} = \langle -2, 6 \rangle$, find the magnitude of \vec{v} .
- Find a vector \vec{v} whose magnitude is 8 and is parallel to the vector $\vec{w} = \langle -3, 5 \rangle$.
- Find the components of a vector whose magnitude is 5 and whose angle with the horizontal is 30° .
- Consider two forces $\vec{F}_1 = \langle 20, 0 \rangle$ and $\vec{F}_2 = 10\langle \cos \theta, \sin \theta \rangle$. Write a function for $\left\| \vec{F}_1 + \vec{F}_2 \right\|$ in terms of θ .
- Three forces with magnitudes of 75 pounds, 100 pounds, and 125 pounds act on an object at angles of 30° , 45° , and 120° , respectively, with the positive x -axis. Find the direction and magnitude of the resultant of these forces.
- Determine whether \vec{v} and \vec{u} are parallel, orthogonal, or neither.
 - $\vec{u} = \langle 5, 6 \rangle$, $\vec{v} = \langle -12, 10 \rangle$.
 - $\vec{u} = \langle 5, 6 \rangle$, $\vec{v} = \langle -20, -24 \rangle$.
 - $\vec{u} = \langle 5, 6 \rangle$, $\vec{v} = \langle -15, 18 \rangle$.

22. Find the domain of the vector valued function $\vec{r}(t) = \sqrt{4-t^2} \vec{i} + t^2 \vec{j}$.
23. Find $\left\| \vec{r}(t) \right\|$ for the vector valued function $\vec{r}(t) = \sin(3t) \vec{i} + \cos(3t) \vec{j}$
24. Evaluate the limit: $\lim_{t \rightarrow 2} \left(t \vec{i} + \frac{t^2 - 4}{t^2 - 2t} \vec{j} \right)$.
25. Determine the intervals of t on which the vector-valued function is continuous: $\vec{r}(t) = t \vec{i} + t^{-2} \vec{j}$.
26. True or False: If f and g are first-degree polynomial functions, then the curve given by $x = f(t)$ and $y = g(t)$ is a line. Is the converse true or false?
27. Find $\vec{r}'(t)$ where $\vec{r}(t) = 6t \vec{i} - 7t^2 \vec{j}$.
28. A baseball is thrown from the stands 32 feet above the field at a launch angle of 30° . Its initial speed is 32 ft/sec.
- Write an acceleration vector and a velocity vector for its motion at any time t .
 - Write a pair of parametric equations to represent its position at any time t .
 - How far from the base of the stands will the ball strike the ground?
29. Given an object moving along a path defined by the vector valued function $\vec{r}(t) = t \vec{i} + t^2 \vec{j}$.
- Find the velocity
 - Find the speed
 - Find the acceleration
 - Find any values for t at which the particle is moving horizontally.
 - Find any value for t at which the particle is stopped.
30. Given an object moving along a path with velocity vector $\vec{v} = \langle -3 \sin t, 3 \cos t \rangle$. At time $t=0$ it is at the point $(4, -1)$.
- Find its position at time $t = \frac{\pi}{2}$.
 - Find the speed at time $t = \frac{\pi}{6}$.
 - Find the acceleration vector.
 - Find any values for t at which the particle is moving horizontally.
 - Find any value for t at which the particle is stopped.
31. Write the equation of the tangent line to $r = 4 \cos \theta$ at $\left(2, \frac{\pi}{3} \right)$.
32. Find all relative maxima and minima:
- $r = 2 - 3 \sin \theta$
 - $r = \cos^2 \theta$

Answers

1. It crosses at time $t = \frac{11}{5}$ at the point $\left(\frac{-7}{5}, \frac{11}{5}\right)$.

2.



3.

4. (23, 1)

5.

6. $\langle 7, 8 \rangle$

7. $2\sqrt{10}$

8. $\frac{\langle -3, 5 \rangle}{\sqrt{(-3)^2 + 5^2}} \cdot 8 = \left\langle \frac{-24}{\sqrt{34}}, \frac{40}{\sqrt{34}} \right\rangle$

9. $\langle 2.5, 2.5\sqrt{3} \rangle$

10. $\|F_1 + F_2\| = 10\sqrt{5 + 4 \cos \theta}$

11. $(75, 30^\circ)$ has rectangular coordinates of $(37.5\sqrt{3}, 37.5)$; $(100, 45^\circ)$ has rectangular coordinates of $(50\sqrt{2}, 50\sqrt{2})$; $(125, 120^\circ)$ has rectangular coordinates of $(-62.5, 62.5\sqrt{3})$. The resultant vector is the sum of the three vectors when added in rectangular form. The resultant vector is $\langle 73.162583, 216.46385 \rangle$. In polar form that is $\langle 228.494, 71.325^\circ \rangle$. So the magnitude is 228.494 pounds in a direction of 71.325° .

12. a.

b.

c.

13. a.

b.

c.

14. $\vec{v} = \langle 1, -4 \rangle$. New position at $t = 19$ is $(-16, 18) + 13(1, -4) = (-3, -34)$.

15. $(x, y) = (9, -4) + (t-3)(5, -2)$ or $x = -6 + 5t$ and $y = 2 - 2t$.

16. Speed is $\sqrt{9t^4 - 14t^2 - 8t + 13}$. Stopped at time $t = 1$

17.

18. a. No, because their x -coordinates are the same at time $t = 4$ and their y -coordinates are the same at time $t = 1$.

b. $k = 1$.

c.

19. a. $t = \frac{\pi}{4}$

b. $t = \frac{\pi}{2}, \frac{3\pi}{2}$

c. Never

d. 1

e. 1.479

20.

21.

22. $-2 \leq t \leq 2$

23. 1

24. $2\vec{i} + 2\vec{j}$

25. $(-\infty, 0) \cup (0, \infty)$

26. True, false

27. $\vec{r}'(t) = 6\vec{i} - 14t\vec{j}$

28. a. $a(t) = \langle 0, -32 \rangle$; $v(t) = \langle 16\sqrt{3}, -32t + 16 \rangle$

b. $x = 16\sqrt{3}t$ and $y = -16t^2 + 16t + 32$

c.

29. a. $\vec{v}(t) = \vec{i} + 2t\vec{j}$

b. speed = $\sqrt{1 + 4t^2}$

c. $\vec{a}(t) = 2\vec{j}$

d. $t = 0$

e. never

30. a. (1, 2)

b. speed = 3

c. $\vec{a}(t) = \langle -3\cos t, -3\sin t \rangle$

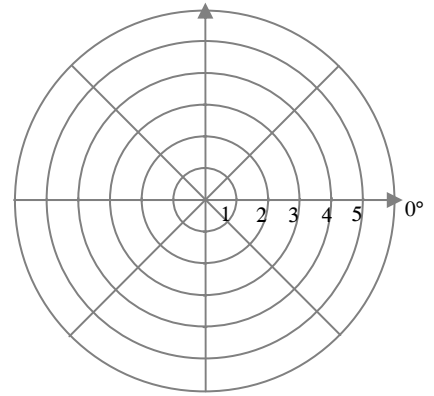
d. $t = \frac{\pi}{2}, \frac{3\pi}{2}$

e. Never

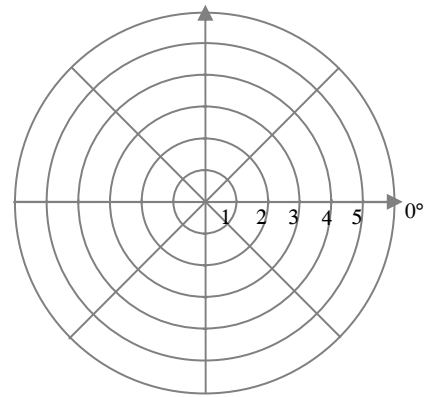
31. $y - \sqrt{3} = \frac{\sqrt{3}}{3}(x - 1)$

32. a. The relative maxima occur when $\sin \theta = \frac{1}{3}$, which is at $\theta = 0.340, 2.802$. The rectangularpoints are $\left(\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$. The relative minima occur when $\cos \theta = 0$, which is at $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$.The rectangular coordinates of the points are $(0, -1)$ and $(0, -5)$.b. The relative extrema occur when $\cos \theta = \pm\sqrt{\frac{2}{3}}$. The two relative maxima are at $\theta = 0.615, 2.526$. The rectangular coordinates of the points are $\left(\frac{2\sqrt{2}}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right)$, and $\left(\frac{-2\sqrt{2}}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right)$. The two relative minima are at $\theta = 0.3757, 5.668$. The rectangularcoordinates of the points are $\left(\frac{2\sqrt{2}}{3\sqrt{3}}, \frac{-2}{3\sqrt{3}}\right)$, and $\left(\frac{-2\sqrt{2}}{3\sqrt{3}}, \frac{-2}{3\sqrt{3}}\right)$.

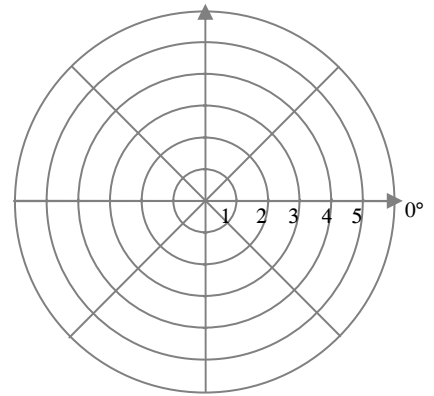
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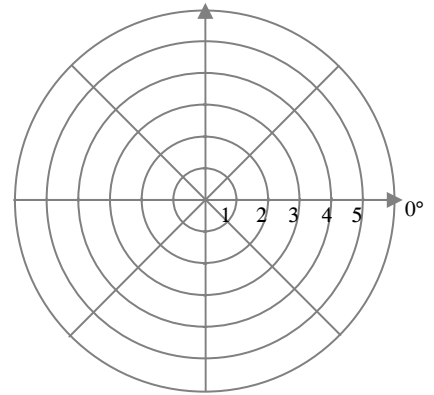
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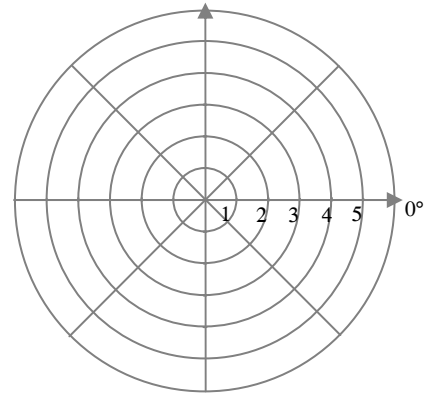
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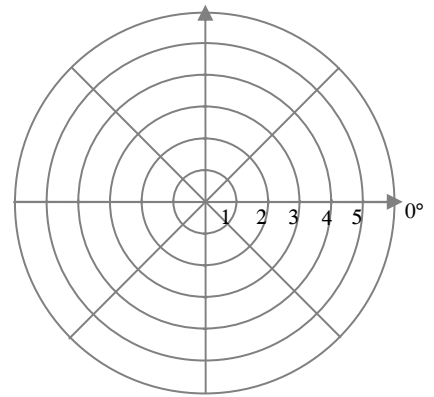
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