AP Calculus BC - Ahlborn | *Name*______________________________

Laboratory Assignment - 7 | *Name*______________________________

*Date turned in*__________________________

Skydiving

In this lab you will investigate the motion of a skydiver in free-fall. If we ignore air resistance and assume that the only force acting upon the skydiver is gravity, which is 9.8 m/sec^2, then we can write a very simple differential equation: $\frac{dv}{dt} = -g$ whose general solution is $v = -gt + C_1$. Since $\frac{dh}{dt} = v$, the height function would be $h(t) = -0.5gt^2 + C_1 t + C_2$.

If there were no air resistance, however, a parachute would be worthless and skydiving would be a once-in-a-lifetime sport. Air resistance produces an upward force that resists the downward motion. Introducing air resistance into the model yields a very different differential equation. In classical Calculus one would not ordinarily try such a problem because the solution is much too complex. But with Euler's Method and the help of a computer to do the calculations, we can readily find a numerical solution which approximates the velocity and height as functions of time before and after the skydiver opens the parachute.

The study of physics has determined that the acceleration of a freely falling body affected by both gravity and air resistance can be expressed as:

$$a(t) = -g + \frac{C \bullet d \bullet A \bullet v^2}{2 \bullet m}$$

where C is the coefficient of air resistance
d is the density of air
A is the surface area of the body
v is the instantaneous velocity
m is the mass of the skydiver
g is the acceleration due to gravity.

We will use C = 0.57, d = 1.3 kg/m^3, and m = 75 kg. The surface area of the skydiver will be A = 0.7 m^2 before the parachute is opened and A = 30 m^2 after the parachute is opened. (A surface area of 30 m^2 for an open parachute is unrealistically small, but a larger number overtaxes the computer system.)

Since $\frac{dv}{dt} = a(t)$, then $\frac{dv}{dt} = -g + \frac{C \bullet d \bullet A \bullet v^2}{2 \bullet m}$

Using this differential equation, you should be able to write the recursive formula used in Euler's Method to approximate the velocity. We will take our initial conditions to be $t_0 = 0$ and $v_0 = 0$.

We would also like to find values for the actual height of the skydiver at any time t. Since velocity is simply the rate of change of height with respect to time we can write $\frac{dh}{dt} = v(t)$. Use this to write the recursive formula needed in Euler's Method to approximate the height. We will assume the plane to be flying at 2000 m. Hence $h_0 = 2000$.

1. Load the StudyWorks template C-LAB-7.MCD and look it over. (*My Computer Public Access Ahlborn Ahlborn-Calculus BC C-LAB-7.MCD*)

   Most of the constants we have already explained. A few things need more explanation.

   The letter i is the counter for the recursive expressions. It starts at zero and continues until the value L is reached. L stands for last iteration. Ultimately, the last iteration should be the one which lands the skydiver safely on the ground. L is one of the values you will be altering as you work with the template.

   The letter r stands for ripcord and represents the iteration where the skydiver pulls the ripcord to open the parachute. Since A changes at this time, the template is set up to automatically change the value of A to correspond with whatever time you want the skydiver to pull the ripcord. Thus, r is a value you will be altering as you work.

   The template also varies the values for Δt. From the initial jump until the rip chord is pulled the change in time will be 0.5 seconds. After the ripcord is pulled, the change in velocity is so dramatic, that a much smaller Δt, $\Delta t = 0.1$, is needed for Euler's method to give a reasonable approximation of the skydiver's behavior.

   Just above the graphs you will see a row of four expressions. You do not enter values for these. Instead these are values which StudyWorks will give you each time the worksheet is calculated. The expression v_L represents the value of v at the last iteration, h_r is the value of h when the ripcord is pulled, h_L is the value of h at the last iteration, and t_r is the value of t when the ripcord is pulled.

2. Enter into the template the appropriate values for the other constants and the functions for velocity and height. Leave L and r blank for the moment. Do NOT press F9 yet.

3. For the first situation, let the skydiver just free-fall for 50 seconds without opening his parachute at all.

   a. Since $\Delta t = 0.5$, then it will take 100 iterations to let 50 seconds pass. Let $L = 100$, and $r = 100$. Now reclaculate. (We do not want him to pull the ripcord at all, but we have to enter some value for r. Making it the last iteration causes it to have no effect.)

   b. Look at the graphs of velocity versus time and height versus time. The blue horizontal line in the height graph represents ground level. Make a printed copy of the screen with the graphs showing. Write a verbal description of each graph. Explain why they have that shape.

   c. What is the terminal velocity reached? (The value of the velocity where its curve levels off is called the terminal velocity.)

   d. At what height is the skydiver after free-falling for 35 seconds?

4. For the next situation, have the skydiver pull the ripcord after free-falling for 30 seconds.

a. That means you will have to set $r = 60$ and adjust L to a larger value. As you look at the graphs you will see if you have made L large enough (or too large) to just land the skydiver on the ground. Adjust the value appropriately.

b. Again make a printed copy of the screen with the graphs showing. Write a verbal explanation for the difference between these graphs and the previous ones.

c. What terminal velocity is reached after the skydiver opens his parachute?

d. How long did it take from when he opened his parachute until he hit the ground?

5. In order for a skydiver to land safely, he must have his parachute open long enough to reach terminal velocity before impact with the ground. A significantly higher velocity upon impact would cause physical injury. In this problem situation, you are to determine how long the skydiver can wait before pulling the ripcord and still land safely.

   a. Adjust the values for r and L until you have the skydiver opening the parachute at the last possible moment to allow him to just attain terminal velocity with his parachute before impact.

   b. Make a printed copy of the screen with the graphs showing your final result.

   c. How long can the skydiver wait before he must pull the ripcord? How far off the ground will he be at that moment?

6. Prepare a written report which includes each of the following:

   a. This assignment paper as a cover to the report. Please fill in your name(s) and the due date. If you had a partner but are turning in separate reports, put the partner's name in parentheses.

   b. An introductory paragraph about the nature and purpose of the assignment.

   c. Answers to all questions in parts 3., 4., and 5.

   d. A summary of your results, interpretations, and conclusions.

   f. All requested print-outs.