DEFINITIONS, THEOREMS, AND FORMULAS – Beyond AP

Chapters e7 and e8 – Integration Applications and Techniques

❖ ***Indicates that the item should be memorized in exact detail. You may be asked to quote it on a quiz or test.***

1. Def. **Area of a Surface of Revolution**: If $y = f(x)$ is a smooth curve on the interval $[a, b]$, then the area of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is $2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$ where $r(x)$ is the distance between the graph of f and the axis of revolution.

2. *Process* **Trigonometric Substitutions**: To integrate a radical expression of the form:

   a. $\sqrt{a^2 - x^2}$, substitute $x = a\sin\theta$.

   b. $\sqrt{x^2 + a^2}$, substitute $x = a\tan\theta$.

   c. $\sqrt{x^2 - a^2}$, substitute $x = a\sec\theta$.

Chapter L – Definition of a Limit

3. Def: ❖**Two-sided limit at a point**: Let f be a function defined on an open interval containing a (except possibly at a) and let L be a real number. $\lim_{x\to a} f(x) = L$ means that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

4. Def: ❖**Limit of positive infinity**: $\lim_{x\to a} f(x) = +\infty$ means that for every $N > 0$ there exists a $\delta > 0$ such that whenever $0 < |x-a| < \delta$, then $f(x) > N$.

5. Def: ❖**Limit of negative infinity**: $\lim_{x\to a} f(x) = -\infty$ means that for every $N < 0$ there exists a $\delta > 0$ such that whenever $0 < |x-a| < \delta$, then $f(x) < N$.

6. Def: ❖**Limit at positive infinity**: $\lim_{x\to +\infty} f(x) = L$ means that for every $\varepsilon > 0$ there exists an $M > 0$ such that whenever $x > M$, then $|f(x) - L| < \varepsilon$.

7. Def: ❖**Limit at negative infinity**: $\lim_{x\to -\infty} f(x) = L$ means that for every $\varepsilon > 0$ there exists an $M < 0$ such that whenever $x < M$, then $|f(x) - L| < \varepsilon$.