REVIEW FOR AP EXAMINATION - DAY 1

Lesson Goal: Review limits, continuity, derivatives, related rates, differentials, implicit differentiation, max/min points, and inflection points.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is

A. 0 B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. 1 E. nonexistent

---

2. If f is a function such that $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true?

A. The limit of $f(x)$ as x approaches 2 does not exist.

B. f is not defined at $x = 2$.

C. The derivative of f at $x = 2$ is 0.

D. f is continuous at $x = 0$.

E. $f(2) = 0$.

---

3. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

A. None B. I only C. III only D. I and III only E. II and III only

---

4. ♦Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

A. 0 only B. 2 only C. 3 only D. 0 and 3 E. 2 and 3

---

5. Let f be the function defined by $f(x)=\begin{cases} x & x>0 \\ x^3 & x\le 0 \end{cases}$. Which of the following statements about f is true?

   A. f is an odd function.

   B. f is discontinuous at $x=0$.

   C. f has a relative maximum.

   D. $f'(0)=0$.

   E. $f'(x)>0$ for $x\neq 0$.

---

6. If $\frac{d}{dx}f(x)=g(x)$ and if $h(x)=x^2$, then $\frac{d}{dx}f(h(x))=$

   A. $g(x^2)$ B. $2xg(x)$ C. $g'(x)$ D. $2xg(x^2)$ E. $x^2g(x^2)$

---

7. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

   A. $\frac{4}{27}$ B. $\frac{4}{9}$ C. $\frac{3}{4}$ D. $\frac{4}{3}$ E. $\frac{16}{9}$

---

8. ♦A tangent line to the graph of the curve $y=x\cos x$ at the point where $x=1$ forms a right triangle with the coordinate axes. The area of the triangle is approximately

   A. 1.13 B. 1.18 C. 1.21 D. 1.25 E. 1.29

---

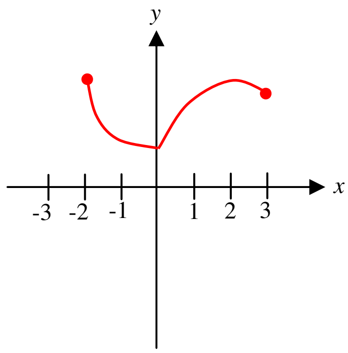
9. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4, 3)$?

   A. $-\frac{25}{27}$ B. $-\frac{7}{27}$ C. $\frac{7}{27}$ D. $\frac{3}{4}$ E. $\frac{25}{27}$

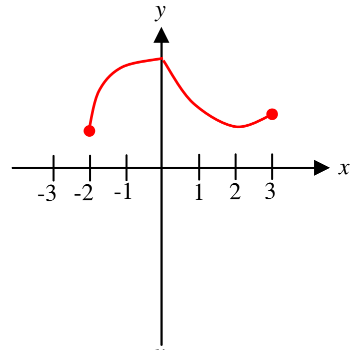
---

10. Let f be a function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?

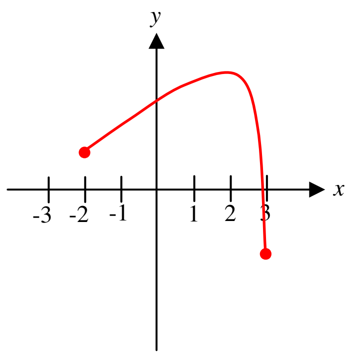
A.



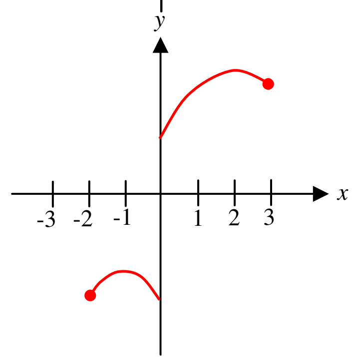
B.



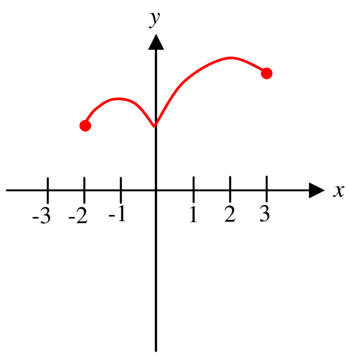
C.



D.



E.



Show all work.

11. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

a. Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

b. On what intervals, if any, is the graph of h concave up? Justify your answer.

c. Write an equation for the line tangent to the graph of h at $x = 4$.

d. Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

DAY 2

Lesson Goal: Review distance-velocity-acceleration, Riemann sums, trapezoidal/midpoint approximations, and Fundamental Theorem of Calculus.

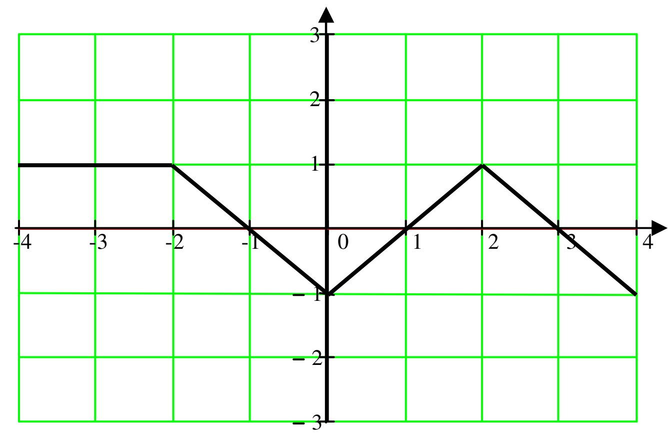
Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The graph of f is shown at the right. Which of the following statements are true?

I. $f(2) > f'(1)$

II. $\int_0^1 f(x)dx > f'(3.5)$

III. $\int_{-1}^1 f(x)dx > \int_{-1}^2 f(x)dx$



A. I only B. II only C. I and II only D. II and III only E. I, II, and III

---

2. ♦An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

A. 2.5 B. 3.5 C. 4.5 D. 5.5 E. 6.5

---

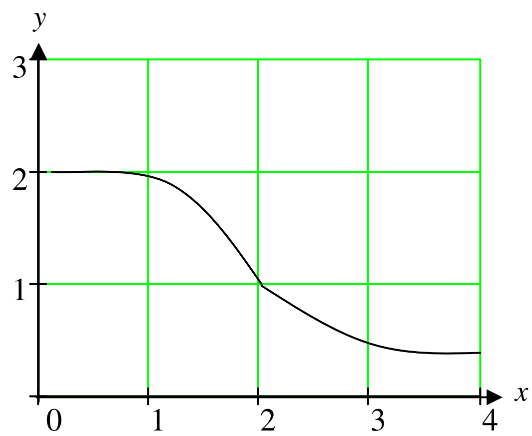
3. A point moves on the x-axis so that its distance from the origin at time t is given by $10t-4t^2$. What is the total distance covered by the point between $t=1$ and $t=2$?

A. 1.0 B. 1.5 C. 2.0 D. 2.5 E. 3.0

---

4. $\frac{d}{dx}\int_0^x \cos(2\pi u)du$ is

A. 0 B. $\frac{1}{2\pi}\sin x$ C. $\frac{1}{2\pi}\cos(2\pi x)$ D. $\cos(2\pi x)$ E. $2\pi\cos(2\pi x)$



5. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

A. 0.3 B. 1.3 C. 3.3 D. 4.3 E. 5.3

---

6. $\lim_{h \to 0} \dfrac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is

A. 0 B. 1 C. 3 D. $2\sqrt{2}$ E. nonexistent

---

7. ♦If $\sin(3x) - 1 = \int_a^x f(t)dt$, then the value of a is

A. 0 B. 1 C. -1 D. $\dfrac{\pi}{3}$ E. $\dfrac{\pi}{6}$

---

8. ♦A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

A. 20 m B. 14 m C. 7 m D. 6 m E. 3 m

---

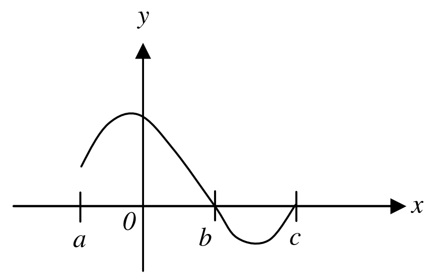
9. Which of the following are true about the function $F(x) = \int_1^x \ln(2t-1)dt$?

I. $F(1) = 0$ II. $F'(1) = 0$ III. $F''(1) = 1$

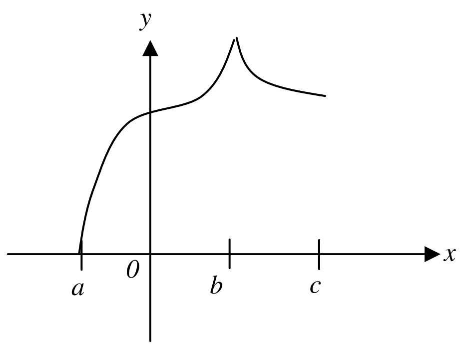
A. I and II only B. I and III only C. II and III only D. I, II, III E. none

---

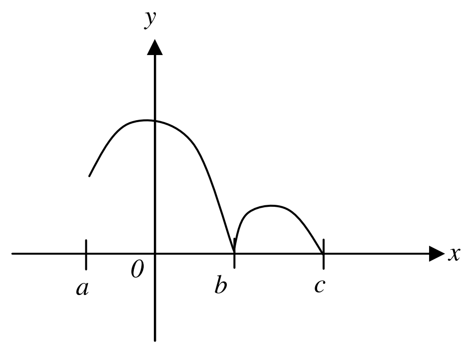
10. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown at right. Which of the following could be the graph of f?



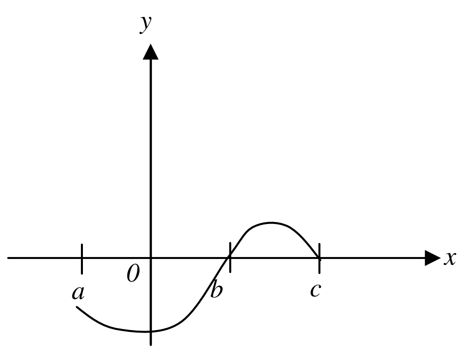
A.



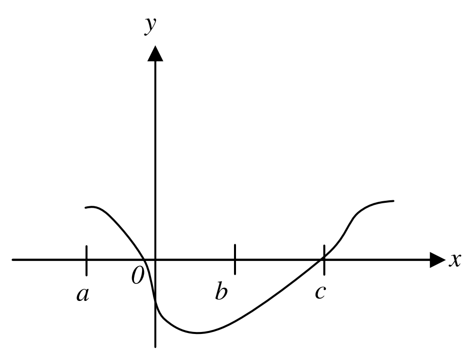
B.



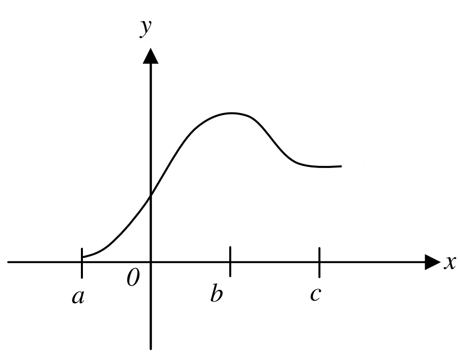
C.



D.

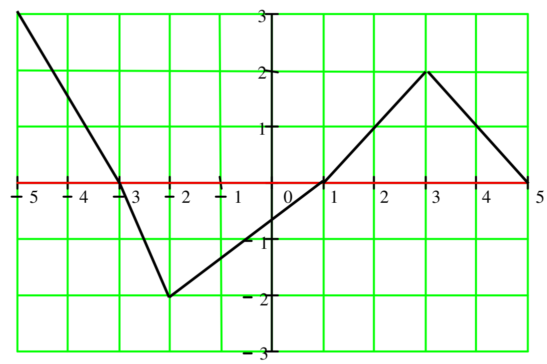


E.



Show all work.

11. Let $G(x) = \int_{-3}^{x} f(t)dt$ and $H(x) = \int_{2}^{x} f(t)dt$ where f is the function graphed at right.



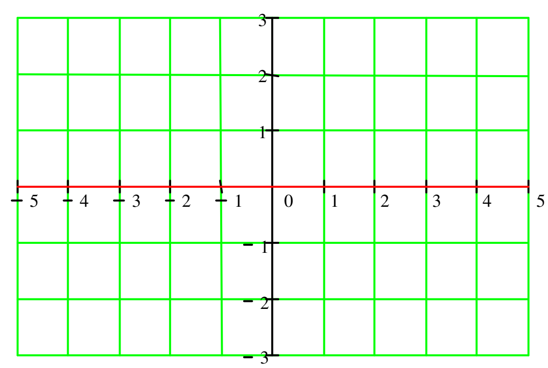
a. How are the values of $G(x)$ and $H(x)$ related? Give a geometric explanation of this relationship.

b. On which subintervals of $[-5, 5]$, if any, is H increasing?

c. At what x-coordinates does G have a relative maximum? Justify your answer.

d. On which subintervals of $[-5, 5]$, if any, is G concave up?

e. Sketch a graph of G on the axes provided.



DAY 3

Lesson Goal: Review total change, area, volume, arc length, and average value.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

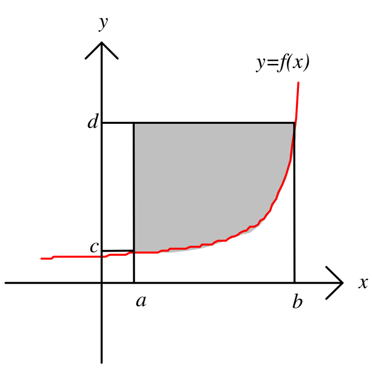
1. ♦Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

   A. 54 gal B. 271 gal C. 865 gal D. 8,647 gal E. 14,778 gal

---

2. The area of the region bounded above by $y = 1 + \sec^2 x$, below by $y = 0$, on the left by $x = 0$ and on the right by $x = \frac{\pi}{4}$ is approximately

   A. 1 unit^2 B. 1.25 units^2 C. 1.5 units^2 D. 1.75 units^2 E. 2 units^2



3. Which of the following represents the area of the shaded region in the figure above?

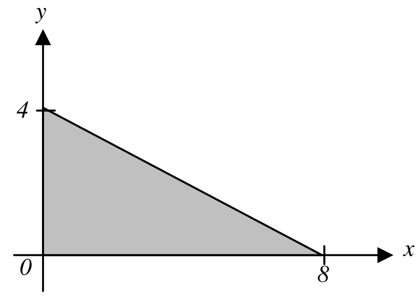
   A. $\int_c^d f(y)\,dy$ B. $\int_a^b (d - f(x))dx$ C. $f'(b) - f'(a)$

   D. $(b-a)[f(b) - f(a)]$ E. $(d-c)[f(b) - f(a)]$

---

4. The average value of the function $f(x) = e^{-2x}$ on the closed interval $[-1, 1]$ is

   A. 0 B. $\frac{1-e^4}{e^2}$ C. $\frac{e^4-1}{e^2}$ D. $\frac{e^4-1}{4e^2}$ E. $\frac{e^4-1}{2e^2}$



5. ♦The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

A. 12.566 B. 14.661 C. 16.755 D. 67.021 E. 134.041

---

6. ♦The region in the first quadrant enclosed by the y-axis and the graphs of $y=\cos x$ and $y=x$ is rotated about the x-axis. The volume of the solid generated is

A. 0.484 B. 0.877 C. 1.520 D. 1.831 E. 3.040

---

7. ♦What is the length of the arc of $y=\frac{2}{3}x^{3/2}$ from $x=0$ to $x=3$?

A. $\frac{8}{3}$ B. 4 C. $\frac{14}{3}$ D. $\frac{16}{3}$ E. 7

---

8. The base of a solid is the region in the first quadrant enclosed by the graph of $y=2-x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

A. $\pi\int_0^2 (2-y)^2\,dy$ B. $\int_0^2 (2-y)\,dy$ C. $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

D. $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$ E. $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

---

9. The volume generated by revolving about the *x*-axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$, for $0 \le x \le 1$, is

A. $\pi \int_0^1 (2x - 2x^2)^2 \, dx$

B. $\pi \int_0^1 (4x^2 - 4x^4) \, dx$

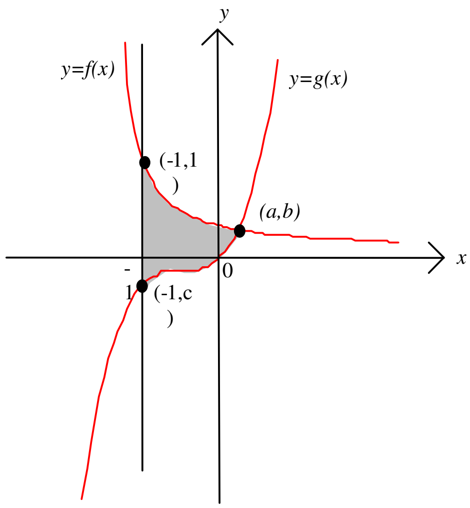
C. $2\pi \int_0^1 x(2x - 2x^2) \, dx$

D. $\pi \int_0^2 \left(\sqrt{\frac{y}{2}} - \frac{y}{2} \right)^2 dy$

E. $\pi \int_0^2 \left(\frac{y}{2} - \frac{y^2}{2} \right) dy$

---

10. The curves $y = f(x)$ and $y = g(x)$ shown in the figure at right intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by



A. $\int_{-1}^{a} \left(|f(x)| - |g(x)| \right) dx$

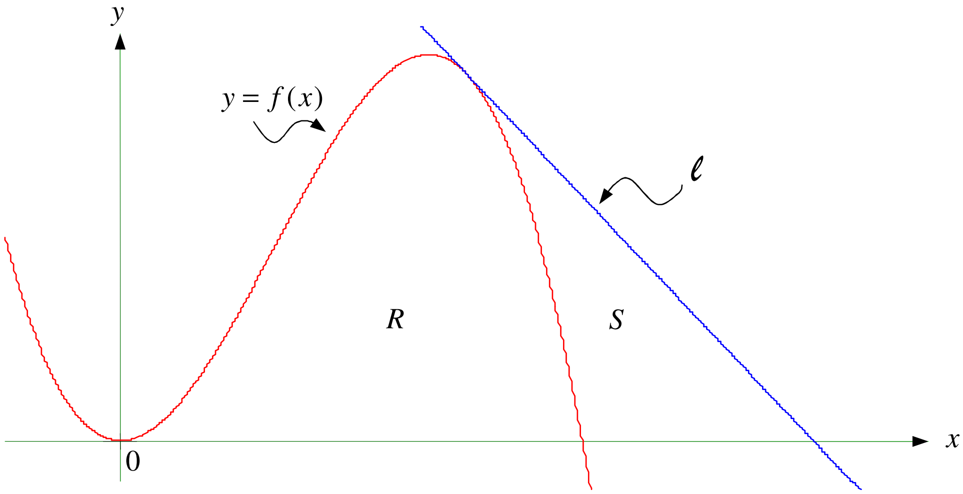
B. $\int_{-1}^{b} g(x) dx + \int_{b}^{c} f(x) dx$

C. $\int_{-1}^{c} \left(f(x) - g(x) \right) dx$

D. $\int_{-1}^{a} \left(f(x) - g(x) \right) dx$

E. $\int_{0}^{a} \left(f(x) - g(x) \right) dx + \int_{-1}^{0} \left(f(x) + g(x) \right) dx$

Show all work. Remember to show the set-up for any calculator-generated answers.



11. ♦Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

a. Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

b. Find the area of S.

c. Find the volume of the solid generated when R is revolved about the x-axis.

DAY 4

Lesson Goal: Review the indefinite integral, substitution, integration by parts, partial fractions, and indefinite integrals.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

A. $2\sqrt{x^3+1}+C$ B. $\frac{3}{2}\sqrt{x^3+1}+C$ C. $\sqrt{x^3+1}+C$

D. $\ln\sqrt{x^3+1}+C$ E. $\ln(x^3+1)+C$

---

2. $\int \sin(2x+3)dx =$

A. $-2\cos(2x+3)+C$ B. $-\cos(2x+3)+C$ C. $\frac{-1}{2}\cos(2x+3)+C$

D. $\frac{1}{2}\cos(2x+3)+C$ E. $\cos(2x+3)+C$

---

3. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}dx =$

A. $\arcsin\frac{x}{5}+C$ B. $\arcsin x+C$ C. $\frac{1}{5}\arcsin\frac{x}{5}+C$

D. $\sqrt{25-x^2}+C$ E. $2\sqrt{25-x^2}+C$

---

4. Given $f(x)=\begin{cases} x+1 & x<0, \\ \cos(\pi x) & x\ge 0, \end{cases}$ $\int_{-1}^{1} f(x)dx =$

A. $\frac{1}{2}+\frac{1}{\pi}$ B. $\frac{-1}{2}$ C. $\frac{1}{2}-\frac{1}{\pi}$ D. $\frac{1}{2}$ E. $\frac{-1}{2}+\pi$

---

5. If $\int x^2 \cos x\,dx = f(x) - \int 2x \sin x\,dx$, then $f(x) =$

A. $2\sin x + 2x\cos x + C$ B. $x^2 \sin x + C$ C. $2x\cos x - x^2 \sin x + C$

D. $4\cos x - 2x\sin x + C$ E. $(2 - x^2)\cos x - 4\sin x + C$

---

6. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

A. $\int_0^{1/2} \sin^2 y\,dy$ B. $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ C. $2\int_0^{\pi/4} \sin^2 y\,dy$

D. $\int_0^{\pi/4} \sin^2 y\,dy$ E. $2\int_0^{\pi/6} \sin^2 y\,dy$

---

7. $\int \arcsin x\,dx =$

A. $\sin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$ B. $\frac{(\arcsin x)^2}{2} + C$ C. $\arcsin x + \int \frac{dx}{\sqrt{1-x^2}}$

D. $x\arccos x - \int \frac{x\,dx}{\sqrt{1-x^2}}$ E. $x\arcsin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

---

8. $\int \frac{dx}{(x-1)(x+2)} =$

A. $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$ B. $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right| + C$ C. $\frac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

D. $(\ln|x-1|)(\ln|x+2|) + C$ E. $\ln\left|(x-1)(x+2)^2\right| + C$

---

9. $\int_4^{\infty} \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ is

A. $7^{2/3}$ B. $\frac{3}{2}\left(7^{2/3}\right)$ C. $9^{2/3}+7^{2/3}$ D. $\frac{3}{2}\left(9^{2/3}+7^{2/3}\right)$ E. nonexistant

---

10. If $f(x)=\begin{cases} 8-x^2 & -2 \le x \le 2, \\ x^2 & \textit{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

A. 0 and 8 B. 8 and 16 C. 16 and 24 D. 24 and 32 E. 32 and 40

---

11. $\int_1^2 \frac{x^2-x}{x^3}\,dx =$

A. $\ln 2-\frac{1}{2}$ B. $\ln 2+\frac{1}{2}$ C. $\frac{1}{2}$ D. 0 E. $\frac{1}{4}$

---

12. If $f(x)=\begin{cases} x & \text{for } x \le 1, \\ \frac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_0^e f(x)\,dx =$

A. 0 B. $\frac{3}{2}$ C. 2 D. e E. $e+\frac{1}{2}$

---

13. $\int xe^{2x}\,dx =$

A. $\frac{xe^{2x}}{2}-\frac{e^{2x}}{4}+C$ B. $\frac{xe^{2x}}{2}-\frac{e^{2x}}{2}+C$ C. $\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+C$

D. $\frac{xe^{2x}}{2}+\frac{e^{2x}}{2}+C$ E. $\frac{x^2e^{2x}}{4}+C$

Show all work.

14. Determine whether or not $\int_0^\infty xe^{-x}dx$ converges. If it converges, give the value. Show your reasoning.

DAY 5

Lesson Goal: Review differential equations, slope fields, logarithmic and exponential functions.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\frac{d}{dx}\left(2^x\right)=$

   A $2^x - 1$ B. $\left(2^{x-1}\right)x$ C. $\left(2^x\right)\ln 2$ D. $\left(2^{x-1}\right)\ln 2$ E. $\frac{2x}{\ln 2}$

---

2. If $f(x)=\left(x^2+1\right)^{2-3x}$, then $f'(1)=$

   A. $\frac{-1}{2}\ln(8e)$ B. $-\ln(8e)$ C. $-\frac{3}{2}\ln(2)-\frac{1}{2}$ D. $\frac{-1}{2}$ E. $\frac{1}{8}$

---

3. ♦Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

   A. 0.048 B. 0.144 C. 5.827 D. 23.308 E. 1,640.250

---

4. Let $f(x)=x^3+x$. If h is the inverse function of f, then $h'(2)=$

   A. $\frac{1}{13}$ B. $\frac{1}{4}$ C. 1 D. 4 E. 13

---

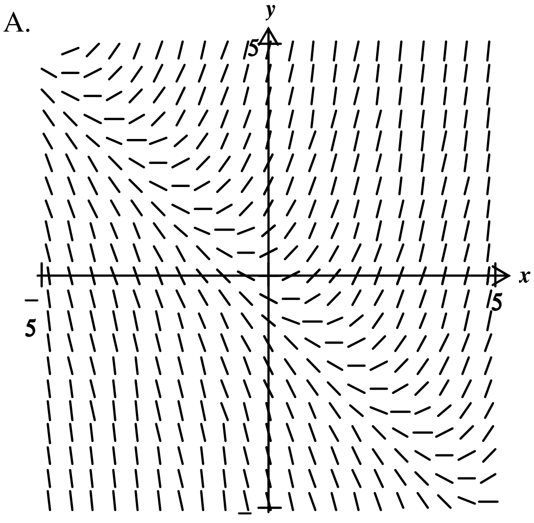
5. If $\frac{dy}{dx}=2y^2$ and if $y=-1$ when $x=1$, then when $x=2$, $y=$

   A. $\frac{-2}{3}$ B. $\frac{-1}{3}$ C. 0 D. $\frac{1}{3}$ E. $\frac{2}{3}$

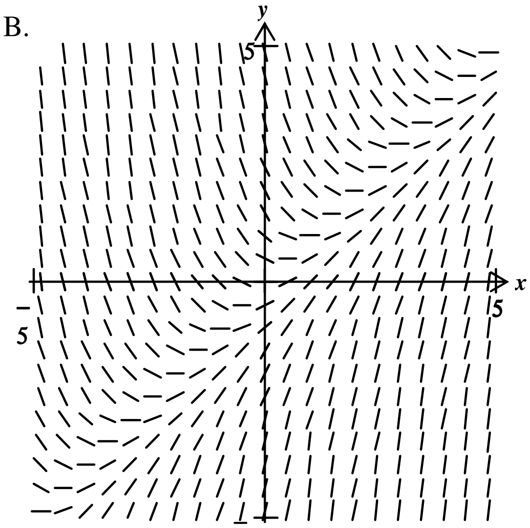
---

6. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

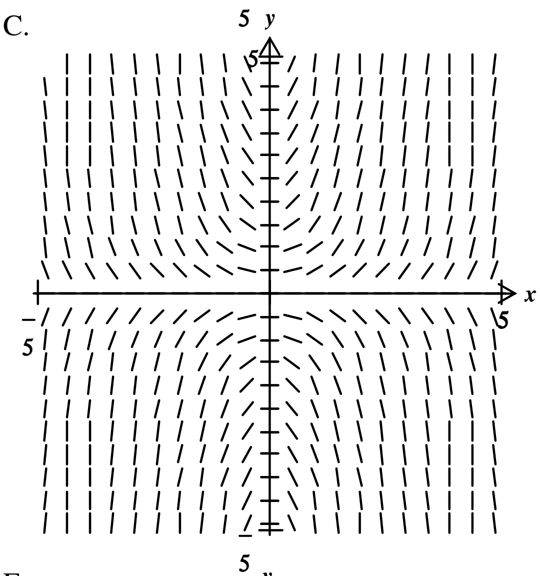
A.



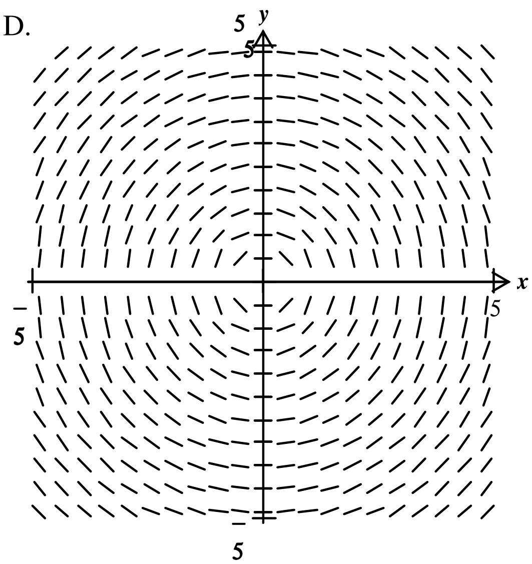
B.



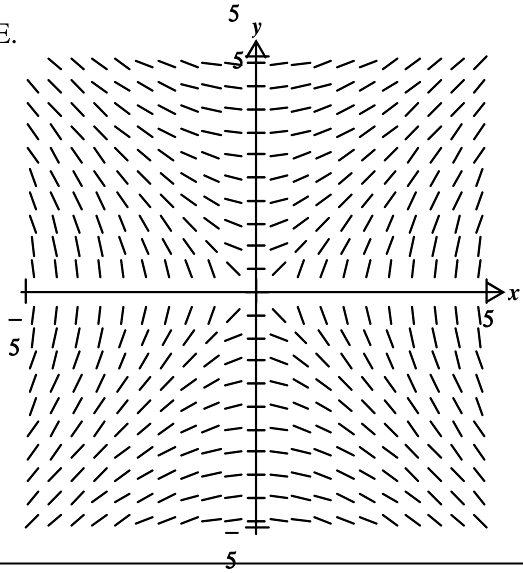
C.



D.



E.



7. ♦A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

A. 4.2 pounds B. 4.6 pounds C. 4.8 pounds D. 5.6 pounds E. 6.5 pounds

---

8. ♦If $f(x) = \dfrac{e^{2x} - \ln x}{2x}$, then $f'(3.05) =$

A. 32.214 B. 120.723 C. 122.225 D. 891.722 E. 1581.738

---

9. ♦Population y grows according to the equation $\dfrac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

A. 0.069 B. 0.200 C. 0.301 D. 3.322 E. 5.000

---

10. ♦If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

A. $\dfrac{e}{2}\left(e^2 + 1\right)$ B. e C. $\dfrac{e^3}{2}$ D. $\dfrac{e}{2}$ E. $\dfrac{\left(e^3 - e\right)}{2}$

Show all work.

11. Given the differential equation $\frac{dy}{dx} = \frac{-xy}{\ln y}$, $y > 0$.

    a. Find the general solution of the differential equation.

    b. Find the solution that satisfies the condition that $y = e^2$ when $x = 0$. Express your answer in the form $y = f(x)$.

    c. Explain why $x = 2$ is not in the domain of the solution found in part (b).

DAY 6

Lesson Goal: Review logistic equations, Euler's method, logarithmic differentiation.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$, and $f(x) > 0$ for all x, then $f(x) =$

   A. $3 + e^{-x^2}$ B. $\sqrt{3} + e^{-x}$ C. $1 + e^{-x}$ D. $\sqrt{3 + e^{-x^2}}$ E. $\sqrt{3 + e^{x^2}}$

---

2. What is $\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x}$?

   A. -1 B. 0 C. 1 D. 2 E. No limit

---

3. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

   A. -5 B. -4.25 C. -4 D. -3.75 E. -3.5

---

4. $\lim_{x \to \infty} (1 + 5e^x)^{1/x}$ is

   A. 0 B. 1 C. e D. e^5 E. nonexistent

---

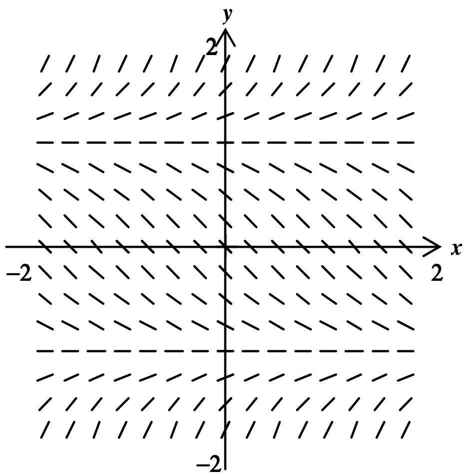
5. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

   A. e^2 B. 1 C. $\frac{1}{2}$ D. 0 E. no limit

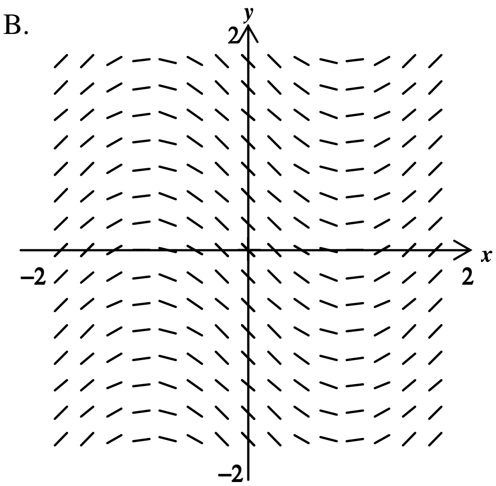
---

6. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

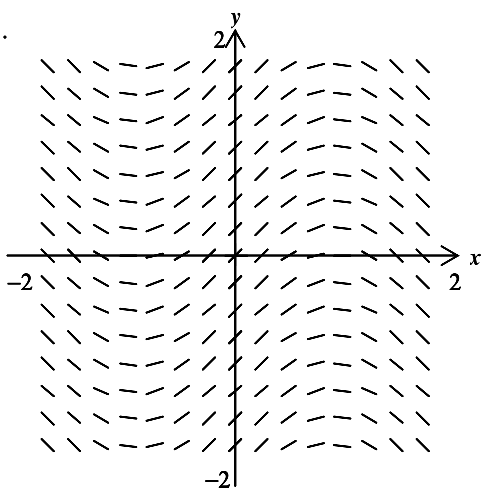
A.



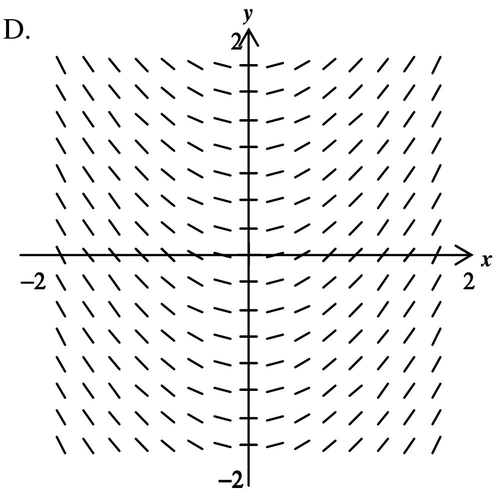
B.



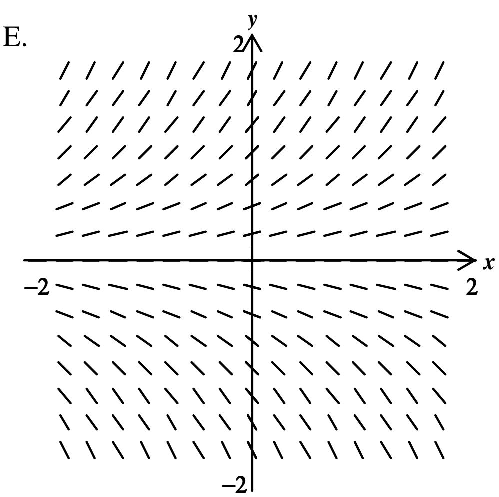
C.

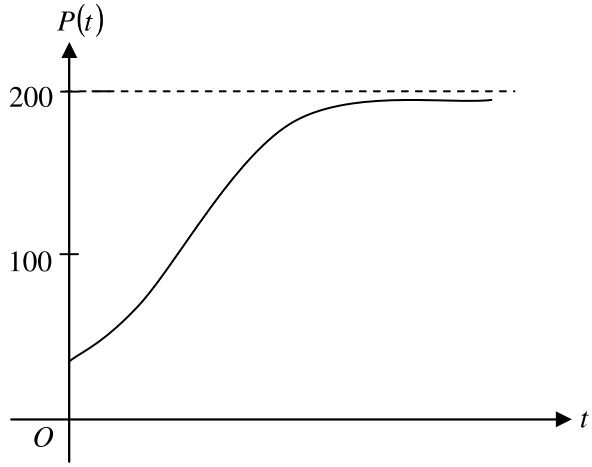


D.



E.





7. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

A. $\frac{dP}{dt} = 0.2P - 0.001P^2$ B. $\frac{dP}{dt} = 0.1P - 0.001P^2$ C. $\frac{dP}{dt} = 0.2P^2 - 0.001P$

D. $\frac{dP}{dt} = 0.1P^2 - 0.001P$ E. $\frac{dP}{dt} = 0.1P^2 + 0.001P$

---

8. If $y = x^{\ln x}$ then y' is

A. $\frac{x^{\ln x} \ln x}{x^2}$ B. $x^{1/x} \ln x$ C. $\frac{2x^{\ln x} \ln x}{x}$ D. $\frac{x^{\ln x} \ln x}{x}$

E. none of the above

Show all work.

9. Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

a. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

b. Find the values of the constants m, b, and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.

c. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

d. Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) = 0$. Find the value of k.

DAY 7

Lesson Goal: Review parametric derivatives, length of a path, and speed/velocity.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

   A. $\frac{e^t}{t}$ B. $\frac{2e^t}{t}$ C. $\frac{e^{|t|}}{t^2}$ D. $\frac{4e^t}{2t-1}$ E. e^t

---

2. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

   A. $(2, \ln 2)$ B. $(e^2, 2)$ C. $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$ D. $(3, \ln 3)$ E. $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

---

3. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is

   A. $(0, -1)$ B. $(0, 12)$ C. $(2, -2)$ D. $(2, 0)$ E. $(2, 8)$

---

4. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

   A. $\int_0^4 \sqrt{4t+1}\, dt$ B. $2\int_0^4 \sqrt{t^2+1}\, dt$ C. $\int_0^4 \sqrt{2t^2+1}\, dt$

   D. $\int_0^4 \sqrt{4t^2+1}\, dt$ E. $2\pi\int_0^4 \sqrt{4t^2+1}\, dt$

---

5. If $x = t^2 + 1$ and $y = t^3$, then $\dfrac{d^2y}{dx^2} =$

A. $\dfrac{3}{4t}$ B. $\dfrac{3}{2t}$ C. $3t$ D. $6t$ E. $\dfrac{3}{2}$

---

6. ♦Consider the curve in the *xy*-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

A. 20.086 B. 0.342 C. -0.005 D. -0.011 E. -0.033

---

7. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

A. -1 only B. 0 only C. 2 only

D. -1 and 2 only E. -1, 0, and 2

---

8. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

A. 0 only B. 1 only C. 0 and $\dfrac{2}{3}$ only

D. 0, $\dfrac{2}{3}$, and 1 E. No value

---

9. In the *xy*-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\dfrac{dx}{dt} > 0$, what is the value of $\dfrac{dy}{dt}$ when the particle is at the point $(2, 2)$?

A. $\dfrac{2}{3}$ B. $\dfrac{2\sqrt{10}}{3}$ C. 3 D. 6 E. $6\sqrt{10}$

---

10. If f is a vector-valued function defined by $f(t)=\left(e^{-t}, \cos t\right)$, then $f''(t)=$

A. $-e^{-t}+\sin t$ B. $e^{-t}-\cos t$ C. $\left(e^{-t}, -\sin t\right)$ D. $\left(e^{-t}, \cos t\right)$ E. $\left(e^{-t}, -\cos t\right)$

---

11. If $x=e^{2t}$ and $y=\sin(2t)$, then $\dfrac{dy}{dx}=$

A. $4e^{2t}\cos(2t)$ B. $\dfrac{e^{2t}}{\cos(2t)}$ C. $\dfrac{\sin(2t)}{2e^{2t}}$ D. $\dfrac{\cos(2t)}{2e^{2t}}$ E. $\dfrac{\cos(2t)}{e^{2t}}$

---

12. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x=t^2+1$ and $y=\ln(2t+3)$, then the acceleration vector is

A. $\left(2t, \dfrac{2}{2t+3}\right)$ B. $\left(2t, \dfrac{-4}{(2t+3)^2}\right)$ C. $\left(2, \dfrac{4}{(2t+3)^2}\right)$

D. $\left(2, \dfrac{2}{(2t+3)^2}\right)$ E. $\left(2, \dfrac{-4}{(2t+3)^2}\right)$

---

13. If a particle moves in the xy-plane so that at time $t>0$ its position vector is $\left(\ln\left(t^2+2t\right), 2t^2\right)$, then at time $t=2$, its velocity vector is

A. $\left(\dfrac{3}{4}, 8\right)$ B. $\left(\dfrac{3}{4}, 4\right)$ C. $\left(\dfrac{1}{8}, 8\right)$ D. $\left(\dfrac{1}{8}, 4\right)$ E. $\left(\dfrac{-5}{16}, 4\right)$

Show all work. Remember to show the set-up for any calculator-generated answers.

14. ♦ An object moving along a curve in the *xy*-plane is at position $(x(t),\ y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

a. Write an equation for the line tangent to the curve at position $(2, -3)$.

b. Find the acceleration vector and the speed of the object at time $t = 1$.

c. Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

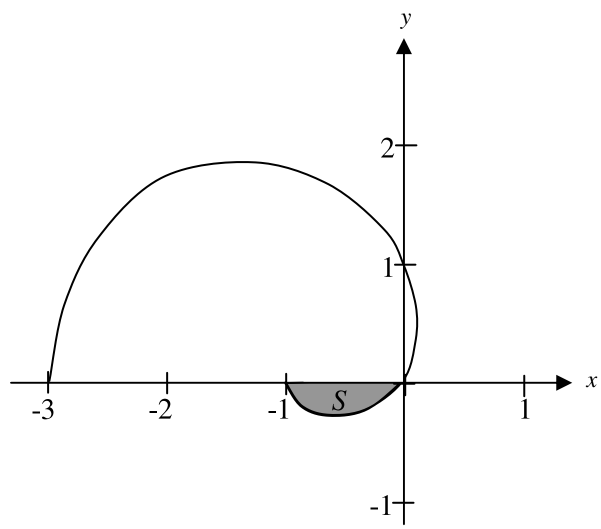
d. Is there a time $t \geq 0$ at which the object is on the *y*-axis? Explain why or why not.

DAY 8

Lesson Goal: Review polar derivatives and polar area.

Show all work.

1. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown at right. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

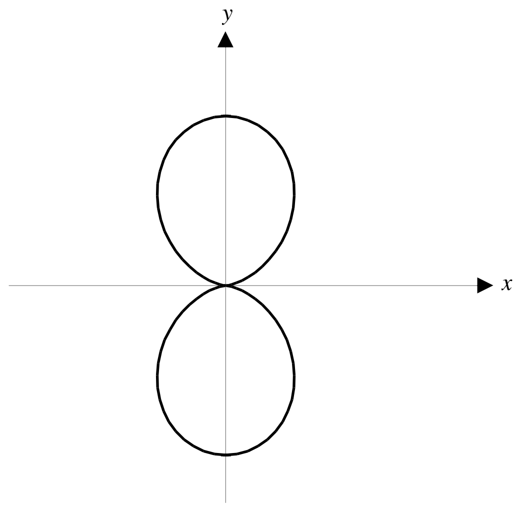


   a. Write an integral expression for the area of S.

   b. Write expressions for $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ in terms of θ.

   c. Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \dfrac{\pi}{2}$. Show the computations that lead to your answer.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.



2. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2 \theta$ shown in the figure above?

A. $\frac{1}{2}\int_0^{\pi} \sin^2 \theta \, d\theta$ B. $\int_0^{\pi} \sin^2 \theta \, d\theta$ C. $\frac{1}{2}\int_0^{\pi} \sin^4 \theta \, d\theta$

D. $\int_0^{\pi} \sin^4 \theta \, d\theta$ E. $2\int_0^{\pi} \sin^4 \theta \, d\theta$

---

3. ♦For all values of x, the continuous function f is positive an decreasing. Let g be the function given by $g(x) = \int_2^x f(t)dt$. Which of the following could be a table of values for g?

A.

x	$g(x)$
1	-2
2	0
3	1

B.

x	$g(x)$
1	-2
2	0
3	3

C.

x	$g(x)$
1	1
2	0
3	-2

D.

x	$g(x)$
1	2
2	0
3	-1

E.

x	$g(x)$
1	3
2	0
3	2

---

4. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

A. $\frac{1}{2}\int_0^{\pi}(4\sin\theta - 2)^2\,d\theta$ B. $\frac{1}{2}\int_{\pi/4}^{3\pi/4}(4\sin\theta - 2)^2\,d\theta$ C. $\frac{1}{2}\int_{\pi/6}^{5\pi/6}(4\sin\theta - 2)^2\,d\theta$

D. $\frac{1}{2}\int_{\pi/6}^{5\pi/6}(16\sin^2\theta - 4)\,d\theta$ E. $\frac{1}{2}\int_0^{\pi}(16\sin^2\theta - 4)\,d\theta$

---

5. Suppose $g'(x) < 0$ for all $x \ge 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \ge 0$. Which of the following statements is FALSE?

A. F takes on negative values.

B. F is continuous for all $x > 0$.

C. $F(x) = x\,g(x) - \int_0^x g(t)\,dt$

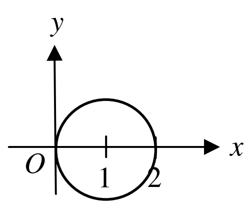
D. $F'(x)$ exists for all $x > 0$.

E. F is an increasing function.

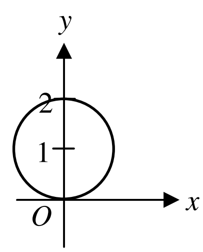
---

6. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

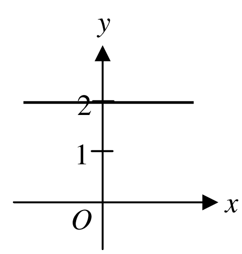
A.



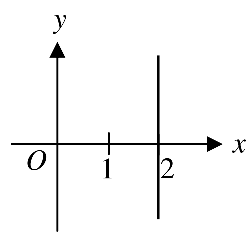
B.



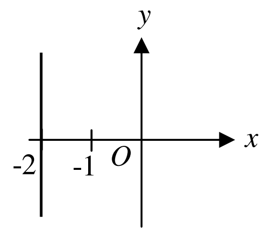
C.



D.



E.



---

7. Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = 0$ and $\lim_{x \to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

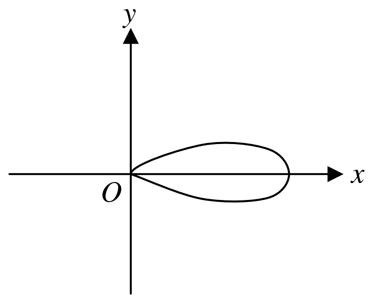
A. 0

B. $\frac{f'(x)}{g'(x)}$

C. $\lim_{x \to 0} \frac{f'(x)}{g'(x)}$

D. $\frac{f'(x)g(x) - f(x)g'(x)}{(f(x))^2}$

E. nonexistent



8. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4\cos(3\theta)$ shown in the figure above?

A. $16 \int_{-\pi/3}^{\pi/3} \cos(3\theta)\, d\theta$

B. $8 \int_{-\pi/6}^{\pi/6} \cos(3\theta)\, d\theta$

C. $8 \int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\, d\theta$

D. $16 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

E. $8 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

DAY 9

Lesson Goal: Review tests for convergent series.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$

II. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^{n}$

III. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{2\ln n}$

A. I only B. II only C. III only

D. I and III only E. I, II, and III

---

2. $\displaystyle\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^{i} =$

A. $\frac{3}{2}-\left(\frac{1}{3}\right)^{n}$ B. $\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^{n}\right]$ C. $\frac{3}{2}\left(\frac{1}{3}\right)^{n}$ D. $\frac{2}{3}\left(\frac{1}{3}\right)^{n}$ E. $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

---

3. ♦Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\displaystyle\sum_{n=1}^{\infty}a_n$ converges to k, which of the following must be true?

A. $\displaystyle\lim_{n\to\infty}a_n = k$ B. $\displaystyle\int_1^n f(x)\,dx = k$ C. $\displaystyle\int_1^{\infty} f(x)\,dx$ diverges

D. $\displaystyle\int_1^{\infty} f(x)\,dx$ converges E. $\displaystyle\int_1^{\infty} f(x)\,dx = k$

---

4. ♦If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

A. $\sum_{n=1}^{\infty} (-1)^n a_n$ converges
B. $\sum_{n=1}^{\infty} (-1)^n b_n$ converges
C. $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges
D. $\sum_{n=1}^{\infty} b_n$ converges
E. $\sum_{n=1}^{\infty} b_n$ diverges

---

5. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \ldots + \frac{(\ln 2)^n}{n!} + \ldots$?

A. $\ln 2$
B. $\ln(1 + \ln 2)$
C. 2
D. e^2
E. The series diverges.

---

6. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

A. $\lim_{n\to\infty} \frac{e}{n!} < 1$
B. $\lim_{n\to\infty} \frac{n!}{e} < 1$
C. $\lim_{n\to\infty} \frac{n+1}{e} < 1$
D. $\lim_{n\to\infty} \frac{e}{n+1} < 1$
E. $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

---

7. Which of the following series converges?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$
II. $\sum_{n=1}^{\infty} \frac{1}{n}$
III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

A. I only
B. III only
C. I and II only
D. I and III only
E. I, II, and III

---

8. What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}\,dx$ converges?

   A. $p < -1$ B. $p > 0$ C. $p > \frac{1}{2}$ D. $p > 1$

   E. There are no values of p for which this integral converges.

---

9. What are all values of p for which the infinite series $\sum_1^\infty \frac{n}{n^p + 1}$ converges?

   A. $p > 0$ B. $p \ge 1$ C. $p > 1$ D. $p \ge 2$ E. $p > 2$

---

10. What is the value of $\sum_{n=1}^\infty \frac{2^{n+1}}{3^n}$?

   A. 1   B. 2   C. 4   D. 6   E. Series diverges

Show all work.

11. Determine all values of x for which the series $\sum_{k=0}^\infty \frac{2^k x^k}{\ln(k+2)}$ converges. Justify your answer.

DAY 10

Lesson Goal: Review intervals of convergence, error bounds, and Taylor polynomials.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

   A. $-1 < x < 1$
   B. $x > 1$ only
   C. $x \geq 1$ only
   D. $x < -1$ and $x > 1$ only
   E. $x \leq -1$ and $x \geq 1$

---

2. ♦Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

   A. -30
   B. -15
   C. -5
   D. $\frac{-5}{6}$
   E. $\frac{-1}{6}$

---

3. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{x^n}{n}$ converges?

   A. $-1 \leq x \leq 1$
   B. $-1 < x \leq 1$
   C. $-1 \leq x < 1$
   D. $-1 < x < 1$
   E. All real x

---

4. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{2n-1}}{2n-1}$, then $f'(x) =$

   A. $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$
   B. $\sum_{n=1}^{\infty}(-1)^{n}x^{2n-2}$
   C. $\sum_{n=1}^{\infty}(-1)^{2n}x^{2n}$
   D. $\sum_{n=1}^{\infty}(-1)^{n}x^{2n}$
   E. $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n}$

---

5. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

   A. $-1 \le x < 1$ B. $-1 \le x \le 1$ C. $0 < x < 2$ D. $0 \le x < 2$ E. $0 \le x \le 2$

---

6. Which of the following series converges for all real numbers x ?

   A. $\sum_{n=1}^{\infty} \frac{x^n}{n}$ B. $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ C. $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ D. $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ E. $\sum_{n=1}^{\infty} \frac{n!\,x^n}{e^n}$

---

7. ♦Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

   A. $2-(x-3)+3(x-3)^2+2(x-3)^3$ B. $2-(x-3)+3(x-3)^2+4(x-3)^3$

   C. $2-(x-3)+6(x-3)^2+12(x-3)^3$ D. $2-x+3x^2+2x^3$

   E. $2-x+6x^2+12x^3$

---

8. If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

   A. $\frac{1}{5}$ B. 1 C. $\frac{5}{4}$ D. $\left(\frac{5}{4}\right)^{100}$

   E. The sequence does not converge

---

9. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

   A. $-3 < x \le 3$ B. $-3 \le x \le 3$ C. $-2 < x < 4$ D. $-2 \le x < 4$ E. $0 \le x \le 2$

Show all work.

10. Let f be the function given by $f(x)=\dfrac{2x}{1+x^2}$.

a. Write the first four nonzero terms and the general term of the Taylor series for f about $x=0$.

b. Does the series found in part (a), when evaluated at $x=1$, converge to $f(1)$? Explain why or why not.

c. The derivative of $\ln(1+x^2)$ is $\dfrac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\ln(1+x^2)$ about $x=0$.

d. Use the series found in part (c) to find a rational number A such that $\left|A-\ln\left(\dfrac{5}{4}\right)\right|<\dfrac{1}{100}$. Justify your answer.

DAY 11

Lesson Goal: Review Taylor and Maclaurin series.

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

A. $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + ...$

B. $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + ...$

C. $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128^7}{7!} + ...$

D. $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + ...$

E. $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128^8}{7!} + ...$

---

2. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

A. $\frac{-1}{6}$ B. 0 C. $\frac{1}{120}$ D. $\frac{1}{6}$ E. 1

---

3. A function has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + ... + \frac{x^{n+3}}{(n+1)!} + ...$. Which of the following is an expression for $f(x)$?

A. $-3x\sin x + 3x^2$ B. $-\cos(x^2) + 1$ C. $-x^2\cos x + x^2$

D. $x^2e^x - x^3 - x^2$ E. $e^{x^2} - x^2 - 1$

---

4. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. 1 D. 3 E. 6

---

5. $\sin(2x) =$

A. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!} + \ldots$

B. $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \ldots$

C. $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n(2x)^{2n}}{(2n)!} + \ldots$

D. $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

E. $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

---

6. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

A. $1 + x^2 + x^4 + x^6 + x^8 + \ldots$

B. $x^2 + x^3 + x^4 + x^5 + \ldots$

C. $x^2 + 2x^3 + 3x^4 + 4x^5 + \ldots$

D. $x^2 + x^4 + x^6 + x^8 + \ldots$

E. $x^2 - x^4 + x^6 - x^8 + \ldots$`

---

7. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

A. $\sin x$ B. $\cos x$ C. e^x D. e^{-x} E. $\ln(1+x)$

---

8. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{3}{2}$ E. $\frac{9}{2}$

Show all work.

9. The function f is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \ldots + (x+1)^n + \ldots = \sum_{n=0}^{\infty} (x+1)^n$$

for all real numbers x for which the series converges.

a. Find the interval of convergence of the power series for *f*. Justify your answer.

b. The power series above is the Taylor series for f about $x = -1$. Find the sum of the series for *f*.

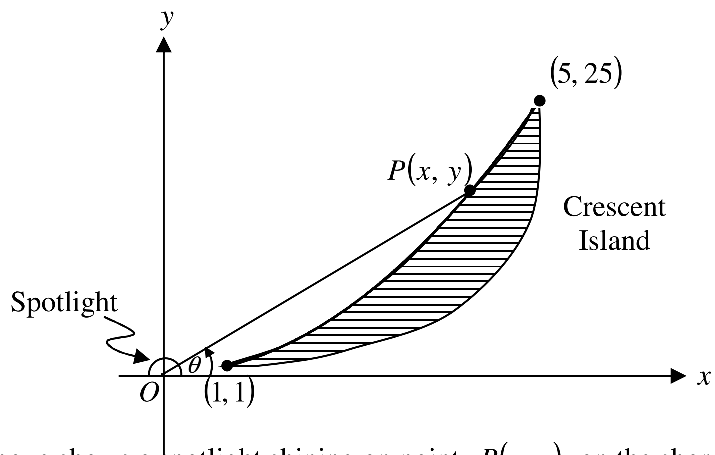
c. Let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the value of $g\left(\frac{-1}{2}\right)$, if it exists, or explain why $g\left(\frac{-1}{2}\right)$ cannot be determined.

d. Let h be the function defined by $h(x) = f(x^2 - 1)$. Find the first three nonzero terms and the general term of the Taylor series for h about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

DAY 12

Lesson Goal: Practice a variety of written problems for the AP Exam.

Questions marked with an ♦ allow use of a calculator. Show all work. Remember to show the set-up for any calculator-generated answers.



1. ♦The figure above shows a spotlight shining on point $P(x, y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point $(1, 1)$ to the point $(5, 25)$. Let θ be the angle between the beam of light and the positive x-axis.

   a. For what values of θ between 0 and 2π does the spotlight shine on the shoreline?

   b. Find the x- and y-coordinates of the point P in terms of $\tan\theta$.

   c. If the spotlight is rotating at the rate of one revolution per minute, how fast is the point P traveling along the shoreline at the instant it is at the point $(3, 9)$?

2. ♦Let f be the function given by $f(x) = e^{-2x^2}$.

a. Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x = 0$.

b. Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.

c. Let g be the function given by the sum of the first four nonzero terms of the power series for $f(x)$ about $x = 0$. Show that $|f(x) - g(x)| < 0.02$ for $-0.6 \le x \le 0.6$.

DAY 13

Lesson Goal: Practice a variety of written problems for the AP Exam.

Questions marked with an ♦ allow use of a calculator. Show all work. Remember to show the set-up for any calculator-generated answers.

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

1. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

   a. Estimate $f'(4)$. Show the work that leads to your answer.

   b. Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

   c. Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

   d. Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

2. ♦The rate at which people enter an amusement park on a given day is modeled by the function E defined by $E(t)=\dfrac{15600}{t^2-24t+160}$. The rate at which people leave the same amusement park on the same day is modeled by the function L defined by $L(t)=\dfrac{9890}{t^2-38t+370}$. Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t=9$, there are no people in the park.

a. How many people have entered the park by 5:00 P.M. $(t=17)$? Round your answer to the nearest whole number.

b. The price of admission to the park is \$15 until 5:00 P.M. $(t=17)$. After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

c. Let $H(t)=\int_9^t \left(E(x)-L(x)\right)dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

d. At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

DAY 14

Lesson Goal: Practice written problems for the AP Exam.

Questions marked with an ♦ allow use of a calculator. Show all work. Remember to show the set-up for any calculator-generated answers.

1. ♦The Taylor series about $x = 0$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 0$ is given by

$$f^{(n)}(0) = \frac{(-1)^{n+1}(n+1)!}{5^n (n-1)^2} \text{ for } n \geq 2.$$

The graph of f has a horizontal tangent line at $x = 0$, and $f(0) = 6$.

a. Determine whether f has a relative maximum, a relative minimum, or neither at $x = 0$. Justify your answer.

b. Write the third-degree Taylor polynomial for f about $x = 0$.

c. Find the radius of convergence of the Taylor series for f about $x = 0$. Show the work that leads to your answer.

2. ♦Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis.

   a. Find the area of the region R.

   b. Find the volume of the solid generated when R is revolved about the x-axis.

   c. The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

Extras

BC 1997 21 area

BC 1985 24 area

BC 1993 45 sum Geometric series

BC 2003 24 who diverges?

BC 1993 Who diverges?

BC 1973 16 build Taylor from known

BC 1969 45 find interval of convergence
