REVIEW FOR FIRST SEMESTER EXAMINATION – DAY 1

Lesson Goal: Review limits, continuity, difference quotients, derivatives, Intermediate and Extreme Value Theorems.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\lim_{x \to 0} (x \csc x)$

A. $-\infty$ B. -1 C. 0 D. 1 E. ∞

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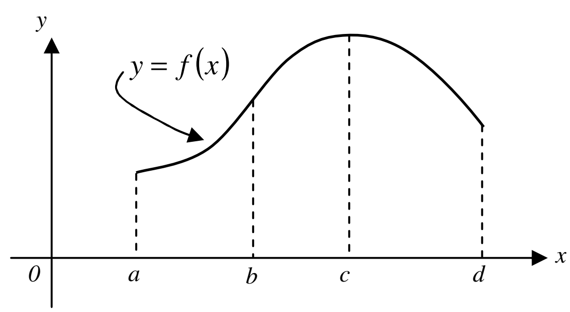
2. The graph of $y = f(x)$ is shown in the figure at right. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$

II. $b < x < c$

III. $c < x < d$



A. I only B. II only C. III only D. I and II E. II and III

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3. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f|x|$ is 5.

II. The maximum value of $|f(x)|$ is 7.

III. The minimum value of $f|x|$ is 0.

A. I only B. II only C. I and II only D. II and III only E. I, II, and III

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4. If $h(x) = [f(x)]^2 - [g(x)]^2$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

   A. 0 B. 1 C. $-4f(x)g(x)$

   D. $[-g(x)]^2 - [f(x)]^2$ E. $-2[-g(x) + f(x)]$

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5. If f is a differentiable function, then $f'(a)$ is given by which of the following?

   I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

   II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

   II. $\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$.

   A. I only B. II only C. I and II only D. I and III only E. I, II, and III

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6. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

   A. $f(0) = 0$.

   B. $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6.

   C. $-1 \le f(x) \le 3$ for all x between -3 and 6.

   D. $f(c) = 1$ for at least one c between -3 and 6.

   E. $f(c) = 0$ for at least one c between -1 and 3.

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7. The function f is defined on all the real numbers such that $f(x)=\begin{cases} x^2+kx-3 & \text{for } x\le 1 \\ 3x+b & \text{for } x>1. \end{cases}$ For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

   A. $k=-1$, $b=-3$ B. $k=1$, $b=3$ C. $k=1$, $b=4$

   D. $k=1$, $b=-4$ E. $k=-1$, $b=6$

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x	0	1	2	3
$f'(x)$	-20	-13	-7	-2

8. Let $f(x)$ be an everywhere differentiable function and $f(3)=-7$. Use the table of values above for the derivative of $f(x)$ to find an approximation for $f(3.5)$.

   A. -9 B. -8 C. -6 D. -3 E. 0

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9. If $f'(x)$ and $g'(x)$ exist and $f'(x)>g'(x)$ for all real x, then the graph of $y=f(x)$ and the graph of $y=g(x)$

   A. do not intersect.

   B. intersect exactly once.

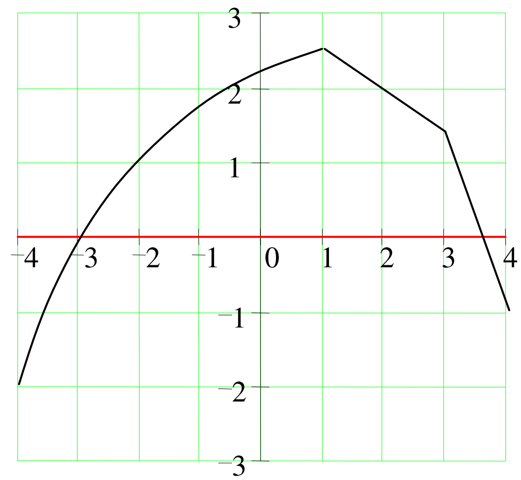
   C. intersect no more than once.

   D. could intersect more than once.

   E. have a common tangent at each point of intersection.

Show all work.

10. The graph of function g is shown at the right.



a. Find $\lim_{x \to 1} g(x)$.

b. Find $\lim_{h \to 0} \dfrac{g(2+h) - g(2)}{h}$.

c. Find $\lim_{x \to -2} g'(x)$.

d. Find $g'\big(g(-3)\big)$.

e. Find $\dfrac{dF}{dx}$ at $x = 2$, if $F(x) = g\big(x^2 - g(x)\big)$.

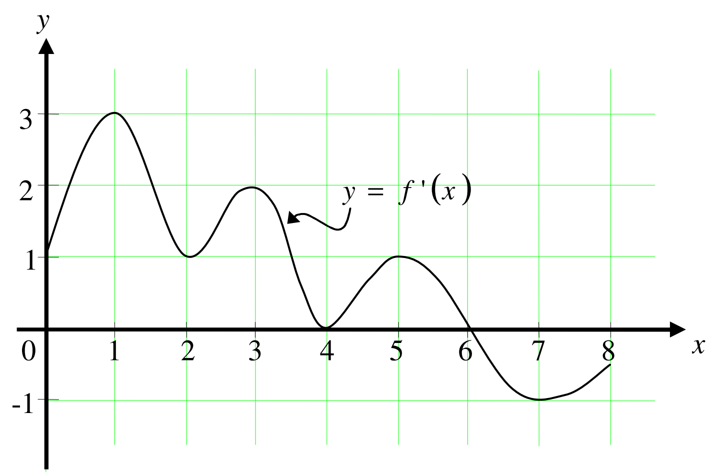
DAY 2

Lesson Goal: Review Mean Value Theorem, optimization, related rates, differentials, implicit differentiation.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$ obtained from a linear approximation to the curve at $x = 0$ is

   A. 2.00 B. 2.03 C. 2.06 D. 2.12 E. 2.24



The function f is defined on the closed interval $[0, 8]$. The graph of its derivative f' is shown above. Questions 2, 3, and 4 refer the above graph.

2. The point $(3, 5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3, 5)$ is

   A. $y = 2$ B. $y = 5$ C. $y - 5 = 2(x - 3)$

   D. $y + 5 = 2(x - 3)$ E. $y + 5 = 2(x + 3)$

3. How many points of inflection does the graph of f have?

   A. two B. three C. four D. five E. six

4. At what value of x does the absolute minimum of f occur?

   A. 0 B. 2 C. 4 D. 6 E. 8

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5. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of the point?

   A. $(1,1)$ B. $(2,1)$ C. $(2,\sqrt{2})$ D. $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$ E. $(3,\sqrt{3})$

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6. The point on the curve of $x^2 + 2y = 0$ that is nearest the point $\left(0,\frac{-1}{2}\right)$ occurs where y is

   A. 2 B. $\frac{1}{2}$ C. 0 D. $\frac{-1}{2}$ E. -1

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7. The graph of $y = 5x^4 - x^5$ has a point of inflection at

   A. $(0,0)$ only B. $(3,162)$ only C. $(4,256)$ only

   D. $(0,0)$ and $(3,162)$ E. $(0,0)$ and $(4,256)$

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8. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

   A. $x < 0$ B. $x < 2$ C. $x < 5$ D. $x > 0$ E. $x > 2$

♦ 9. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

   A. 5 in^3/sec B. 10 in^3/sec C. 15 in^3/sec D. 20 in^3/sec E. 25 in^3/sec

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Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 10. Consider the curve defined by $x^2 + xy + y^2 = 27$.

a. Write an expression for the slope of the curve at any point (x, y).

b. Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

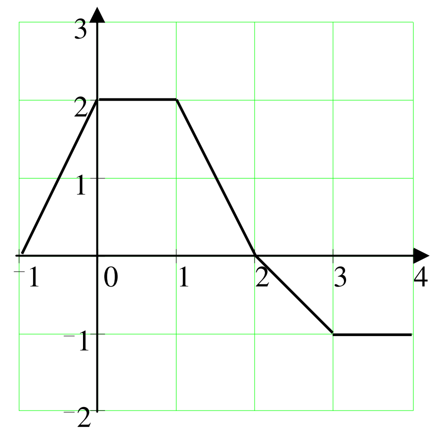
c. Find the points on the curve where the lines tangent to the curve are vertical.

DAY 3

Lesson Goal: Review Riemann sums, trapezoidal/midpoint rules, Fundamental Theorem of Calculus.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown at right. What is the value of $\int_{-1}^{4} f(x)dx$?



A. 1 B. 2.5 C. 4
D. 5.5 E. 8

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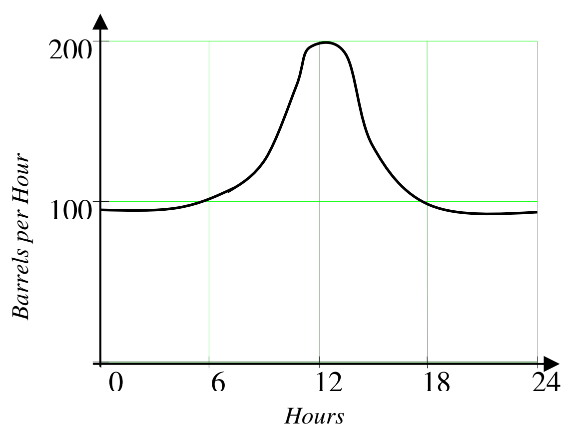
2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown at right. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?



A. 500 B. 600 C. 2,400
D. 3,000 E. 4,800

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3. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)\,dx =$

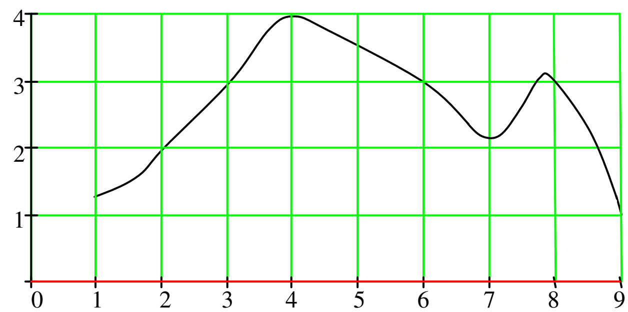
A. 0 B. 1 C. $\dfrac{ab}{2}$ D. $b-a$ E. $\dfrac{b^2-a^2}{2}$

♦ 4. Suppose a car is moving with increasing speed according to the following table. The closest approximation of the distance traveled in the first 10 seconds is

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

A. 150 ft B. 250 ft C. 350 ft D. 450 ft E. 550 ft

♦ 5. The graph of f over the interval $[1, 9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)dx$.



A. 20 B. 21 C. 22 D. 23 E. 24

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x	2	5	7	8
$f(x)$	10	30	40	20

6. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

A. 110 B. 130 C. 160 D. 190 E. 210

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7. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

A. -3 B. -2 C. 2 D. 3 E. 18

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8. The closed interval $[a, b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, x_2, ..., x_n$ where $a = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\Delta x$?

A. $\frac{2}{3}\left(b^{3/2} - a^{3/2}\right)$ B. $b^{3/2} - a^{3/2}$ C. $\frac{3}{2}\left(b^{3/2} - a^{3/2}\right)$

D. $b^{1/2} - a^{1/2}$ E. $2\left(b^{1/2} - a^{1/2}\right)$

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9. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

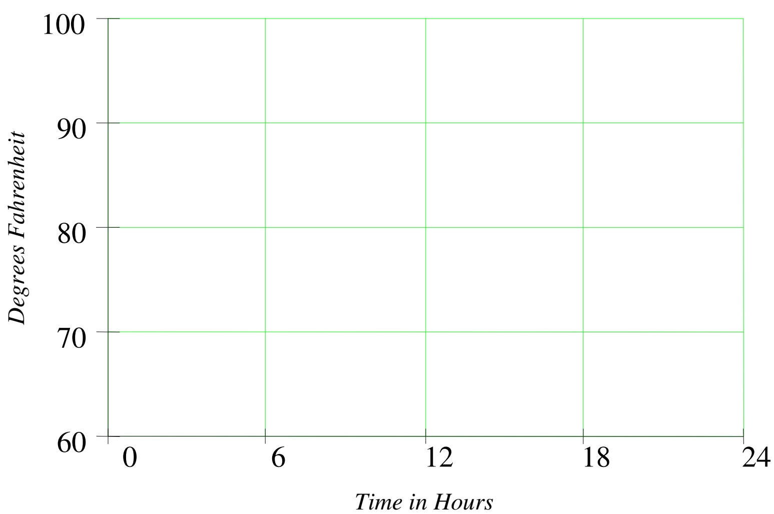
A. $2F(3) - 2F(1)$ B. $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ C. $2F(6) - 2F(2)$

D. $F(6) - F(2)$ E. $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 10. The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right)$, $0 \le t \le 24$, where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

a. Sketch the graph of F on the grid below.



b. Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

c. An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

c. The cost of cooling the house accumulates at the rate of $\$0.05$ per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

DAY 4

Lesson Goal: Review transcendental functions, integration properties and skills.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

   A. $\frac{2\pi}{3}$ B. $\frac{3\pi}{4}$ C. $\frac{5\pi}{6}$ D. π E. $\frac{3\pi}{2}$

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2. $\int \frac{x^3 - 1}{x^2}\, dx =$

   A. $\frac{1}{2}x^2 - \frac{1}{3x^3} + C$ B. $\frac{\frac{1}{4}x^4 - x}{\frac{1}{3}x^3} + C$ C. $1 + \frac{1}{x^3} + C$

   D. $\frac{1}{2}x^2 + \frac{1}{3x^3} + C$ E. $\frac{1}{2}x^2 + \frac{1}{x} + C$

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3. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\, dx =$

   A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{1}{2}\ln 2$ E. $-\ln 2$

♦ 4. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2 - e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

   A. 125 B. 100 C. 88 D. 50 E. 12

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5. $\int \frac{4x}{1+x^2}\,dx =$

A. $4\,\text{Arctan}\,x + C$ B. $\frac{4}{x}\,\text{Arctan}\,x + C$ C. $\frac{1}{2}\ln\left(1+x^2\right)+C$

D. $2\ln\left(1+x^2\right)+C$ E. $2x^2 + 4\ln|x| + C$

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6. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

A. $\int_1^2 \frac{1-u^2}{u}\,du$ B. $\int_2^4 \frac{1-u^2}{u}\,du$ C. $\int_1^2 \frac{1-u^2}{2u}\,du$

D. $\int_1^2 \frac{1-u^2}{4u}\,du$ E. $\int_2^4 \frac{1-u^2}{2u}\,du$

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7. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)dx = \left[\int_a^b f(x)dx\right]\left[\int_a^b g(x)dx\right]$

II. $\int_a^b \left(f(x)+g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

A. I only B. II only C. III only D. II and III only E. I, II, and III

♦ 8. If $f(x) = 2x + \sin x$ and the function g is the inverse of f, then $g'(2) =$

A. 0.32 B. 0.34 C. 0.36 D. 0.38 E. 0.30

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9. What are all values for k for which $\int_{-3}^{k} x^2\,dx = 0$?

A. -3 B. 0 C. 3 D. -3 and 3 E. -3, 0, and 3

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 10. Let $F(x) = \int_0^x \sin\left(t^2\right)dt$ for $0 \le x \le 3$.

a. Use the trapezoidal rule with four equal subdivisions of the closed interval $[0,1]$ to approximate $F(1)$.

b. On what intervals is F increasing?

c. If the average rate of change of F on the closed interval $[1,3]$ is k, find $\int_1^3 \sin\left(t^2\right)dt$ in terms of k.

DAY 5

Lesson Goal: Review differential equations, Euler's Method, slope fields, logistic curves, partial fractions.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{DM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

   A. 50 B. 200 C. 500 D. 1000 E. 2000

♦ 2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's Method is used, starting at $x = 1$ with a step size of 0.5?

   A. 3 B. 5 C. 6 D. 10 E. 12

♦ 3. Let $F(x)$ be an antiderivative of $\frac{2-x}{x^3}$. If $F(1) = 2$, then $F(9) =$

   A. 1.986 B. 2.099 C. 10 D. 17.111 E. -240.333

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4. For $0 < x < \frac{\pi}{3}$, if $y = x^{\cos x}$, then $\frac{dy}{dx} =$

   A. $-(\sin x)x^{(\cos x)-1}$ B. $-(\sin x)(\cos x)x^{(\cos x)-1}$ C. $-(\sin x)(\ln x)x^{\cos x}$

   D. $(\sin x)(\ln x)x^{\cos x}$ E. $x^{\cos x}\left(\frac{\cos x}{x} - (\sin x)(\ln x)\right)$

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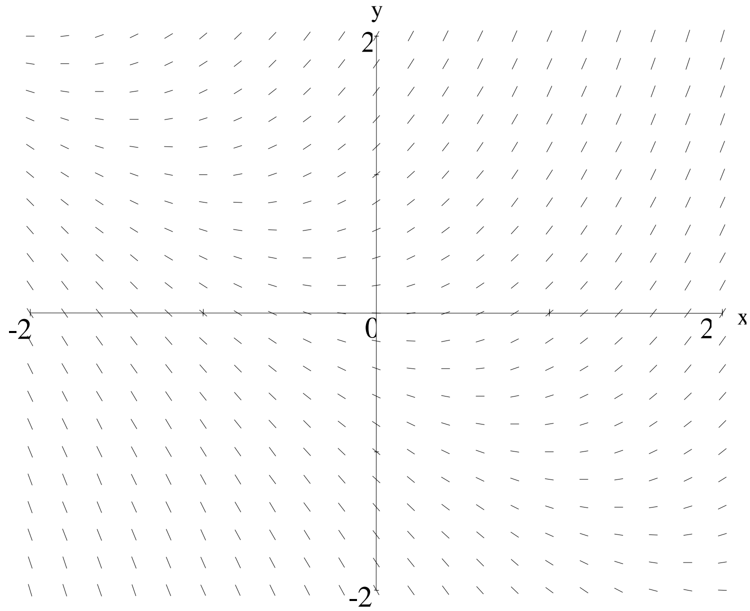
5. If $\frac{dy}{dt} = kt$ and k is a nonzero constant, then y could be

   A. kt^2 B. $\frac{k}{2}t^2 - 8$ C. $2kt^2$ D. $\frac{1}{2}k^2t + 1$ E. $kt - 5$



6. Shown above is a slope field for which of the following differential equations?

A. $\frac{dy}{dx} = 1 + x$ B. $\frac{dy}{dx} = x^2$ C. $\frac{dy}{dx} = x + y$ D. $\frac{dy}{dx} = \frac{x}{y}$ E. $\frac{dy}{dx} = \ln y$

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7. A solution of the equation $\frac{dy}{dx} + 2xy = 0$ that contains the point $(0, e)$ is

A. $y = e^{1-x^2}$ B. $y = e^{1+x^2}$ C. $y = e^{1-x}$ D. $y = e^{1+x}$ E. $y = e^{x^2}$

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8. $\int \frac{1}{(x-1)(x+3)}\, dx =$

A. $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$ B. $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$ C. $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

D. $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$ E. $\ln\left|(x-1)(x+3)\right| + C$

Show all work. Remember to show the set-up for any calculator-generated answers.

9. Let f be the function given by $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$.

   a. Using the definition of the derivative, prove that f is differentiable at $x=0$.

   b. Find $f'(x)$ for $x\neq 0$.

   c. Show that f' is not continuous at $x=0$.

♦ 10. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

   a. Write an equation for y, the amount of oil remaining in the well at any time t.

   b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

   c. In order not to lose money, at what time t should oil no longer be pumped from the well?

DAY 6

Lesson Goal: Review area, volume, arc length, average value.

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The length of a curve from $x=0$ to $x=4$ is given by $\int_0^4 \sqrt{4x^2+1}\,dx$. If the curve contains the point $(3,7)$, which of the following could be an equation for this curve?

   A. $y=2x+1$ B. $y=-x^2-2$ C. $y=x^2-2$

   D. $y=x^2+2$ E. $y=\frac{4}{3}x^3+x-32$

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2. The point on the curve $2y=x^2$ nearest to $(4,1)$ is

   A. $(0,0)$ B. $(2,2)$ C. $(\sqrt{2},1)$ D. $(2\sqrt{2},4)$ E. $(4,8)$

♦ 3. If $0\le k<\frac{\pi}{2}$ and the area under the curve $y=\cos x$ from $x=k$ to $x=\frac{\pi}{2}$ is 0.1, then $k=$

   A. 1.471 B. 1.414 C. 1.277 D. 1.120 E. 0.436

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4. What is the average value of $y=x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

   A. $\frac{26}{9}$ B. $\frac{52}{9}$ C. $\frac{26}{3}$ D. $\frac{52}{3}$ E. 24

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5. The base of a solid is the region enclosed by the graph of $y=e^{-x}$, the coordinate axes, and the line $x=3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

   A. $\frac{(1-e^{-6})}{2}$ B. $\frac{1}{2}e^{-6}$ C. e^{-6} D. e^{-3} E. $1-e^{-3}$

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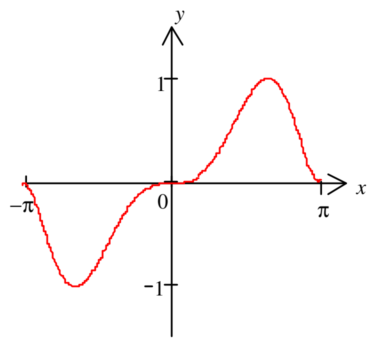
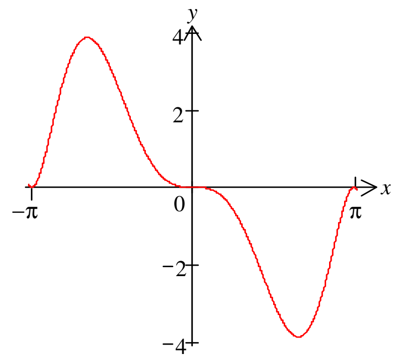
6. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

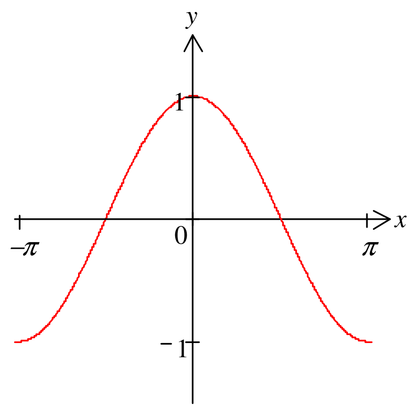
A.



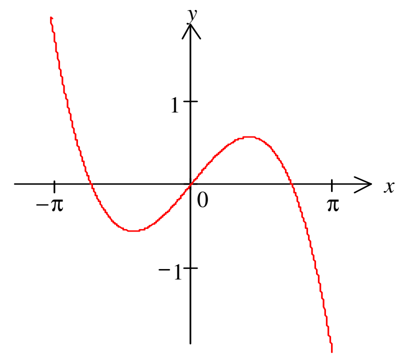
B.



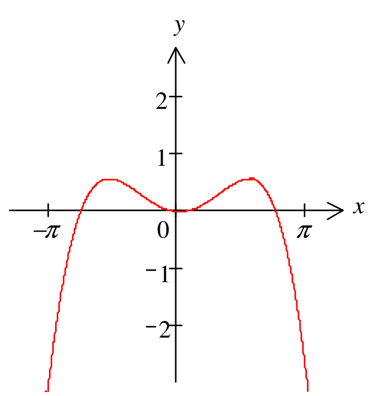
C.



D.



E.



7. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

A. $\dfrac{\pi^2}{4}$ B. $\pi - 1$ C. π D. 2π E. $\dfrac{8\pi}{3}$

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8. Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $y = 2$, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

A. $\pi\int_0^7 (x+1)^{2/3}dx$ B. $\pi\int_1^2 \left(y^3 - 1\right)^2 dy$ C. $\pi\int_1^2 \left(y^3 - 1\right)dy$

D. $\pi\int_0^7 \left(4 - (x+1)^{2/3}\right)dx$ E. $\pi\int_1^2 y^2 dy$

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 9. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

a. Find the area of the region R.

b. Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

c. Find the volume of the solid of revolution formed when R is rotated about the line $y = -2$.
