CLASSWORK 8 – Integration Techniques

Integration Review

Lesson Goal: Select a successful method of integration for each of a wide variety of functions.

1. $\int 5^x dx =$

2. $\int \frac{3\sec^2 x}{\tan x} dx =$

3. $\int \frac{dx}{1+12x^2} =$

4. $\int \frac{2x+1}{x^2+1} dx =$

5. $\int \cot^3 x \csc^3 x dx =$

6. $\int \frac{2x+10}{x^2+4x+4} dx =$

7. $\int_{-1}^{2} \frac{x^3}{x+5} dx =$

8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x \ln(\tan x)}{\tan x} dx =$

9. $\int_{5}^{8} \frac{x}{\sqrt{x-4}} dx =$

Integration by Parts

Lesson Goal: Use integration by parts to rewrite an integral.

10. $\int x^7 \sqrt{5-x^4}\,dx =$

11. $\int x \operatorname{arcsec} x\,dx =$ for $x > 0$

12. $\int 9x\cot^2(3x)\,dx =$

13. $\int \operatorname{arccot} x\,dx =$

14. $\int_1^e x^2 \ln x\,dx =$

Lesson Goal: Use repeated applications of integration by parts to evaluate an integral.

15. $\int x^2 e^x \, dx =$

16. $\int e^{-x} \cos x dx =$

17. $\int \sec^3 x dx =$

18. $\int_0^{\pi/4} \cos^2 x \, dx =$

Review of Limits

Lesson Goal: Use L'Hopital's Rule to evaluate an indeterminate limit in the form of a quotient or product.

19. $\lim_{x \to 0} \frac{\sin x}{e^x} =$

20. $\lim_{x \to 2} \frac{x-2}{x^2-4} =$

21. $\lim_{x \to \infty} \frac{2x-6}{3x+4}$

22. $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} =$

23. Use a graph and table on your TI-CAS to find $\lim_{x\to\infty} x-\sqrt{x^2+x} =$

L'Hopital's Rule

24. $\lim_{x\to\infty}\frac{x}{e^x} =$

25. $\lim_{x\to o^+}\frac{1-\cos\sqrt{x}}{\sin x} =$

26. $\lim_{x\to 0^+}\left[2x^2\ln x\right] =$

27. $\lim_{x\to 1^+}\left[\frac{1}{x-1}-\frac{2}{x^2-1}\right] =$

28. $\lim_{x\to\infty}\frac{3^x}{9^x} =$

29. $\lim_{x\to 0^+}\frac{e^x}{x} =$

30. $\lim_{x\to\infty}\left[x^{-2}\left(e^x-1\right)\right] =$

31. $\lim_{x\to 1}\frac{x^2-1}{x-1} =$

32. $\lim_{x \to 0} \frac{\sin x - x\cos x}{x - \sin x} =$

Lesson Goal: Use L'Hopital's Rule to evaluate an indeterminate limit in the form of an exponential expression.

33. $\lim_{x \to 1} (\ln x)^{x-1} =$

34. $\lim_{x \to \infty} \left(x^2\right)^{\frac{1}{x}} =$

35. $\lim_{x \to 0} (1 - x)^{\frac{1}{x}} =$

36. $\lim_{x \to 0^+} x^{x^2} =$

37. $\lim_{x \to \infty} \left(1 + x^{-2}\right)^x =$

38. $\lim_{x \to \infty} \left(1 + e^{-x}\right)^{\frac{1}{x}} =$

39. $\lim_{x \to o^+} x^{\frac{1}{\ln x}} =$

40. $\lim_{x \to 0^+} (-\ln x)^x =$

Improper Integrals

Lesson Goal: Evaluate an improper integral with one or two infinite integration limits.

41. $\int_1^\infty \frac{1}{x}\,dx$

42. $\int_{-\infty}^{-1} \frac{1}{x^2}\,dx$

43. $\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$

44. $\int_0^\infty te^{-t}\,dt$

45. $\int_0^\infty t^2e^{-t}\,dt$

46. Use TI-CAS: $\int_0^\infty t^4e^{-t}\,dt$

47. Find the value of $1.5!$ by calculating $\int_0^{\infty} t^{1.5}e^{-t}\,dt$ on the TI-CAS.

48. Now use the previous result to calculate WITHOUT AN INTEGRAL the value of $2.5!$. (Hint: Think about the relationship between $8!$ and $9!$.)

49. The factorial function allows us to define $x!$ for any real number. Find the value of $\pi!$ by calculating $\int_0^{\infty} t^{\pi}e^{-t}\,dt$ on your TI-CAS.

Lesson Goal: Evaluate an improper integral with an endpoint or interior-point discontinuity.

50. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

51. $\int_{-5}^{-3} \frac{1}{x+3}\,dx$

52. $\int_{-\pi/4}^{\pi/4} \csc^2 x\,dx$

53. Profits at a small company grow quickly for the first 5 months, then more slowly over the following months. The function $r(t)=\begin{cases} t^2-2 & 0\le t<5 \\ 3t+4 & 5\le t\le 12 \end{cases}$ gives the rate of change in their profits over a twelve-month period. Find the amount of profit over this twelve-month period.

54. An object moves along a number line with velocity $v(t)=\begin{cases} t^2-t & 0\le t<1 \\ \dfrac{5\ln t}{t^2} & 1\le< t<\infty \end{cases}$. Its position at any time t is described by the function $x(t)$. If the object is at $x=7$ when $t=0$, will it ever get to $x=12$?