CLASSWORK 5 – Transcendental Functions

Lesson Goal: Differentiate the natural logarithm function.

Thm. **Derivative of the Natural Logarithm**: If $y = \ln x$, then $y' = \frac{1}{x}$.

Proof:

$f'(x) = \lim_{h \to 0}$ _____ Definition of the derivative

$f'(x) = \lim_{h \to 0} \frac{\ln\left(\quad\right)}{h}$ Property of Logarithms

$f'(x) == \lim_{h \to 0} \left[\frac{1}{x} \frac{x}{h} \ln\left(\frac{x+h}{x} \right) \right]$ Properties of Fractions

$f'(x) = \lim_{h \to 0} \left[\frac{1}{x} \ln\left(1 + \frac{h}{x}\right)^{\frac{x}{h}} \right]$ _____

$f'(x) = \lim_{h \to 0} \frac{1}{x} \bullet \lim_{h \to 0} \left[\ln\left(1 + \frac{h}{x}\right)^{\frac{x}{h}} \right]$ _____

$f'(x) = \frac{1}{x} \bullet \lim_{h \to 0} \left[\ln\left(1 + \frac{h}{x}\right)^{\frac{x}{h}} \right]$ _____

$f'(x) = \frac{1}{x} \bullet \ln \left[\lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{\frac{x}{h}} \right]$ _____

$f'(x) = \frac{1}{x} \bullet \ln(\quad)$ _____

$f'(x) =$ _____

Logarithmic Functions - Differentiate.

1. $y = \ln\left(3x^2\right)$

2. $y = \cos(\ln x)$

3. $y = \ln\left(3x^3 - 3x + 1\right)$

4. $y = \ln(\ln(2x))$

5. $y = \ln\sqrt{4 - 2x}$

6. $y = \ln\left(\dfrac{x-2}{x^2+6x}\right)$

7. $y = \dfrac{x^2 - 3}{1 - 2x}$

8. $y = \sqrt{\dfrac{3-x}{x^2+2}}$

9. Find all relative extrema of the graph of $y = (x-2)\ln x$.

10. Find a linear approximation to the curve $y = x + \ln(x^2 - 1)$ at $x = \sqrt{2}$.

Logarithmic Functions - Integrate

Lesson Goal: Use the logarithm function to integrate a rational function.

11. Differentiate: $y = \ln|x|$.

12. $\int \frac{1}{x-2}dx =$

13. $\int \frac{x^2}{4-x^3}dx =$

14. $\int \frac{(\ln x)^3}{x}dx =$

15. $\int \frac{x^2+1}{4x^3+12x}dx =$

16. $\int \frac{2x-5}{x}dx =$

17. $\int \frac{x^2-2x-1}{x+2}dx =$

18. $\int \frac{2x^2+\ln x}{x}dx =$

19. $\int \frac{x-2}{(x+3)^2}dx$

20. Find the area between the curve of $y = \dfrac{2x}{4 - x^2}$ and the x-axis from $-1 \le x \le 1$.

21. Find the average value of the function $y = \dfrac{3x^2 - 1}{x}$ on the interval $[2, 4]$.

Trigonometric Functions - Integrate.

Lesson Goal: Integrate all six trigonometric functions.

22. $\int \tan(2x)\, dx =$

23. $\int x\sec(x^2 - 1)\, dx =$

24. $\int \csc^2(x)\cot(x)\, dx =$

25. $\int \dfrac{\cos\theta}{\sqrt{\sin\theta}} d\theta =$

26. $\int \left(\cos^2 x - \sin^2 x\right) dx =$

27. $\int \dfrac{\sec(3x)}{\cot(3x)} dx =$

28. $\int \tan^2 x\, dx =$

29. $\int_{\frac{2\pi}{3}}^{\frac{5\pi}{6}} \csc(x) dx =$

30. Find the average value of the function $y = \dfrac{1}{\sin^2 x}$ on the interval $\dfrac{\pi}{4} \le x \le \dfrac{\pi}{2}$.

Inverse Functions

Lesson Goal: Find the derivative of an inverse function.

31. Find the inverse of $y = x^3 - 5$.

32. Show that the function $y = x^3 + 5x - 3$ is strictly monotonic.

33. Given the function $f(x) = x^3 + 5x - 3$, which we previously showed was strictly monotonic, find the slope of f^{-1} at $x = 4$.

34. If $f(x) = x^4 - 3x^2 + 5$, find all points on $f^{-1}(x)$ where the slope is $\dfrac{1}{20}$.

Exponential Functions – Differentiate.

Lesson Goal: Differentiate and integrate the natural exponential function.

35. $y = e^{x^2}$

36. $y = 3x^2 e^{4x}$

37. $y = 2 \bullet \sin(2x) - x \bullet e^{-x}$

38. $y = \ln\left(e^x + e^{-x}\right)$

39. $y = e^{\tan x}$

Exponential Functions - Integrate.

40. $\int e^{-3x}dx =$

41. $\int x^2 e^{2x^3}dx =$

42. $\int e^{\csc x}\csc x\cot x dx =$

43. $\int \frac{e^x}{e^x+1}dx =$

44. $\int \frac{2e^x+e^{-x}}{2e^x-e^{-x}}dx =$

45. $\int \frac{\sin x}{e^{2\cos x}}dx =$

46. $\int \frac{xe^{x^2}}{e^{x^2}+1}dx =$

47. Find all relative extrema and points of inflection of the graph of $y = x^2e^x$. Sketch the graph.

General Exponential and Exponential Functions

Lesson Goal: Differentiate the general exponential and logarithmic function.

48. Differentiate: $y = 3^{4x}$.

49. Differentiate: $y = 2 \bullet 5^x - x \bullet e^{-x}$.

50. Differentiate: $y = (\ln 6)^x$.

51. $\int 6^{5x}dx =$

52. Differentiate: $y = x^{3x}$.

53. Differentiate: $y = x^{x^2}$.

54. Differentiate: $y = \log_7(3x^2)$.

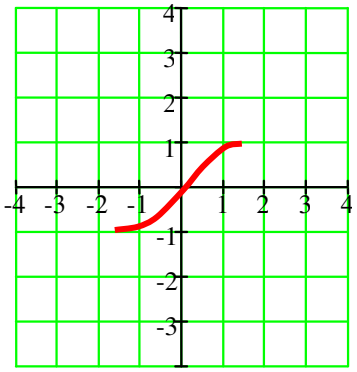
55. Differentiate: $y = \log_3(\sin x)$.

56. Find all points of inflection of the curve $y = x + x(\ln x)^2$.

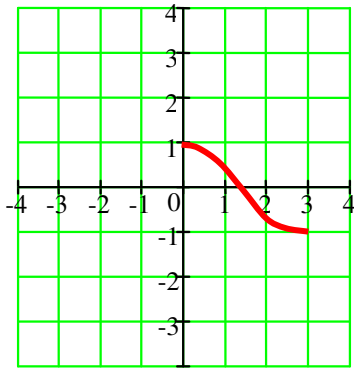
Inverse Trigonometric Functions

Lesson Goal: Differentiate the inverse sine, inverse cosine, inverse tangent and inverse cotangent functions.

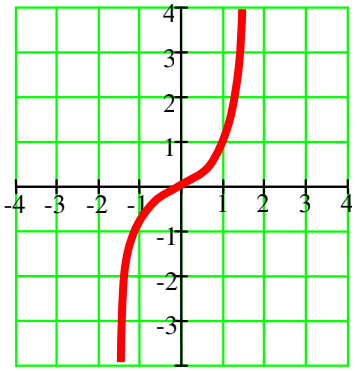
57. Graph the inverse of $y = \sin x$.



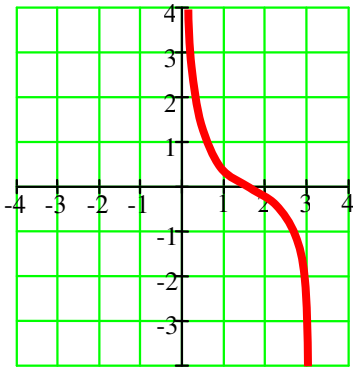
58. Graph the inverse of $y = \cos x$.



59. Graph the inverse of $y = \tan x$.



60. Graph the inverse of $y = \cot x$.



61. Evaluate: $\arccos 0.7 =$

62. Evaluate: $\operatorname{arccot} 2.5 =$

63. Simplify: $\sin\left(\arctan\frac{x}{5}\right) =$

64. Simplify: $\tan\left(\arccos\frac{\sqrt{2}}{3x}\right) =$

65. Differentiate: $y = \arccos(4x)$

66. Differentiate: $y = x^2 \arcsin(2x)$

67. Differentiate: $y = \arcsin(3\sin x)$

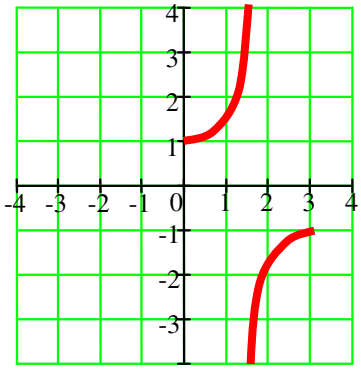
68. Write the linear approximation for $y = x + \arccos x$ at $x = 1$.

69. Differentiate: $y = \arctan\frac{1}{x}$

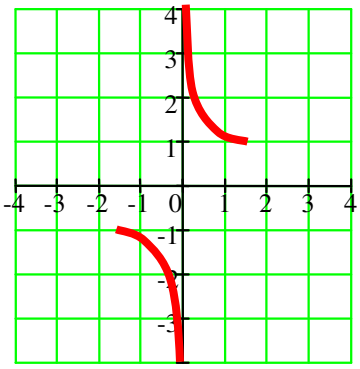
70. Write the linear approximation to the curve $y = \operatorname{arccot} x$ at $x = \frac{\pi}{2}$.

Lesson Goal: Differentiate the inverse secant and inverse cosecant functions.

71. Graph the inverse of $y = \sec x$



72. Graph the inverse of $y = \csc x$



73. Differentiate: $y = \operatorname{arcsec}(2x^2)$.

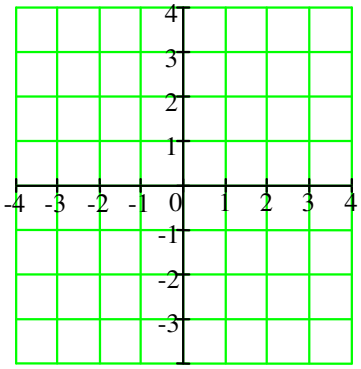
74. Differentiate: $y = 2\operatorname{arccsc}\frac{1}{x}$.

75. Differentiate: $y = \cot(\arcsin x)$.

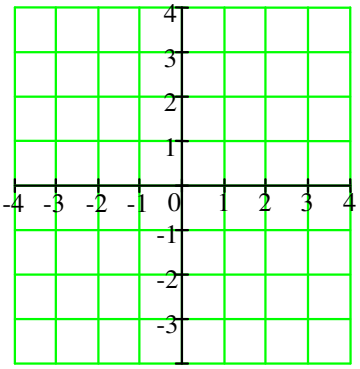
76. Write the linear approximation for $y = \operatorname{arccsc}\sqrt{5}x$ at the point $x = \dfrac{-2}{\sqrt{10}}$.

77. Find all relative extrema and points of inflection for $y = \arccos x^2$.

78. Graph $y = -\arcsin(2x)$.



79. Graph $y = -3\operatorname{arcsec}\dfrac{x}{4}$.



Evaluate.

Lesson Goal: Integrate radical and rational expressions using inverse trigonometric functions.

80. $\displaystyle\int \frac{1}{1+16x^2}dx =$

81. $\displaystyle\int \frac{dx}{x\sqrt{x^2-4}} =$

82. $\int \frac{xdx}{\sqrt{1-x^4}} =$

83. $\int \frac{4x}{\sqrt{1-2x^2}} dx =$

84. $\int \frac{dx}{(x+4)\sqrt{(x+4)^2-1}} =$

85. $\int \frac{dx}{1+(x-3)^2} =$

86. $\int \frac{\arcsin 2x dx}{\sqrt{1-4x^2}} =$

87. $\int \frac{dx}{x\sqrt{x^4-1}} =$

Lesson Goal: Integrate a variety of types of functions using inverse trigonometric functions.

88. $\int \frac{\sin 2x dx}{1+\cos^2 x} =$ compare with $\int \frac{\sin x dx}{1+\cos^2 x} =$

89. $\int \frac{\sec^2 x \tan x}{\sqrt{1-\tan^2 x}} dx =$ compare with $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx =$

90. $\int \frac{e^x dx}{\sqrt{1-e^{2x}}} =$ compare with $\int \frac{e^{2x} dx}{\sqrt{1-e^{2x}}} =$

91. $\int \frac{x-3}{x^2+1} dx =$

92. $\int \frac{\sin 2x}{1-\cos^2 2x} dx =$

93. $\int \frac{\sin 2x}{1+\cos 2x} dx =$

94. $\int \frac{\sec^2 x}{\tan x \sqrt{\tan^2 x - 1}} dx =$

95. $\int_0^{1/2} \frac{-x}{\sqrt{1-x^2}} dx =$

96. $\int_0^{1/2} \frac{x \arcsin^3 x^2}{\sqrt{1-x^4}} dx =$

97. Find the area of the region bounded by $y = \frac{1}{4+x^2}$ where $-2 \le x \le 2$.