SEMESTER I REVIEW

You may use your TI-89 for all questions. All questions are multiple choice except # 18.

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1. If $h(x) = f(x) \bullet g(x)$ where $f(x) = 8x$, then the *slope* of the tangent to $h(x)$ equals zero at $x = 1$ provided that any one of the following conditions is met:

   I. $g(x) = \frac{4}{x}$.

   II. $g(x) = 0$.

   III. $g(x) = x - 2$.

   A. I only B. III only C. I and II only D. II and III only E. I, II, and III

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2. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = x^3 + k$ for every real value of k?

   A. $y = -x^3 + k$ B. $y = \frac{-1}{3x}$ C. $y = \frac{1}{3x}$ D. $\frac{-1}{3x^3} + k$ E. $\frac{-1}{3x^2}$

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3. If $y = (3x^2 - 5x)^6$, then $y' =$

   A. $7(3x^2 - 5x)^7$ B. $6(6x - 5)^5$ C. $6(3x^2 - 5x)$

   D. $6(3x^2 - 5x)^5$ E. $6(3x^2 - 5x)^5(6x - 5)$

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4. An equation line for a tangent to the graph of $yx = 3$ at $x = 3$ is

   A. $y - 1 = \frac{-1}{3}(x - 3)$ B. $y - 3 = \frac{-1}{3}(x - 1)$ C. $y - 1 = \frac{-3}{x^2}(x - 3)$

   D. $y - 3 = \frac{-3}{x^2}(x - 1)$ E. $y - 1 = -3(x - 3)$

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5. The approximate value of $y = \sqrt{4+x}$ at $x = 0.08$ obtained from the tangent to the graph at $x = 0$ is

A. 2.01 B. 2.02 C. 2.03 D. 2.04 E. 2.05

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6. A relative maximum of the function $f(x) = \dfrac{x-2}{x^2}$ occurs at each of the following values for *x:*

A. 0 B. 0, 4 C. 4 D. -4, 0, 4 E. none

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7. The function $g(x) = \int_0^x \sin t\,dt$ has an inflection point at $x =$

A. 0 B. $\dfrac{\pi}{3}$ C. $\dfrac{\pi}{2}$ D. $\dfrac{2\pi}{3}$ E. π

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8. If $2xy^2 - 6y = 5x$, then $\dfrac{dy}{dx} =$

A. $\dfrac{5}{4y-6}$ B. $\dfrac{4xy-5}{6}$ C. $\dfrac{5-2y^2}{4y-6}$ D. $\dfrac{5-2y^2}{4xy-6}$ E. $\dfrac{5-4y}{-6}$

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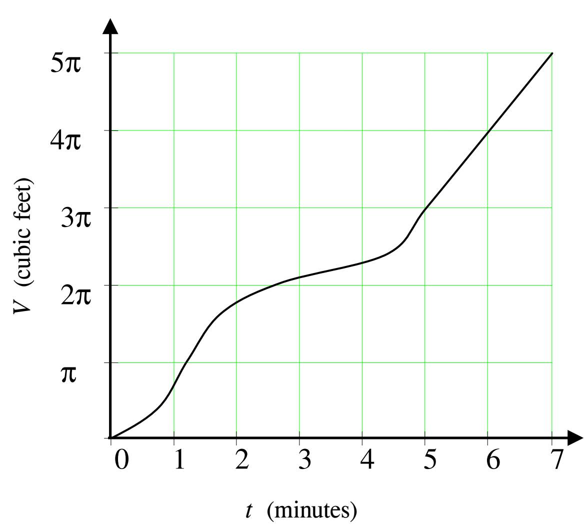
9. Water is dripping into a cylindrical tank of radius 4 feet. The function V whose graph is sketched in the figure gives the volume of water in the tank, measured in cubic feet, after t minutes. At what approximate rate is the height of the water in the tank changing after 6 minutes.



A. 0.04 ft/min

B. 0.06 ft/min

C. 0.08 ft/min

D. 0.10 ft/min

E. 0.12 ft/min

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10. The acceleration of a particle at time t moving along the x-axis is given by $a(t) = 6t$. At the instant when $t = 0$, the particle is at the point $x = 2$ moving with velocity $v = -2$. The position of the particle at $t = \frac{1}{2}$ is

A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$ E. $\frac{9}{8}$

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11. The sale of lumber S (in millions of square feet) for the years 1980 to 1990 is modeled by the function $S(t) = \frac{1}{200}(t^3 - 10t^2 - 16t - 660)$ where t is time in years with $t = 0$ corresponding to the *beginning* of 1980. Determine the year when lumber sales were increasing at the greatest rate.

A. 1982 B. 1983 C. 1984 D. 1985 E. 1990

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12. An approximation for $\int_{-1}^{2} e^{1.5x}\,dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

A. 25.6 B. 25.7 C. 25.8 D. 25.9 E. 30

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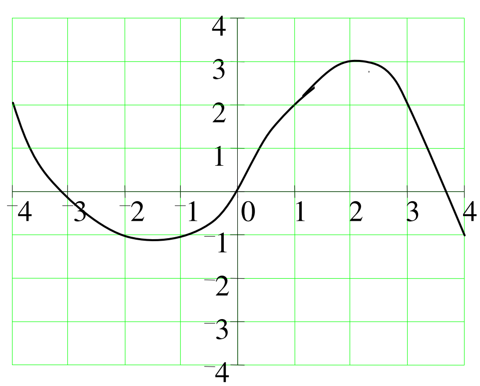
13. The graph of a function f whose domain is the interval $[-4, 4]$ is shown in the figure. Which of the following statements are true?



I. The midpoint approximation of the area under the curve from $x = 0$ to $x = 4$ using only two subdivisions is 8.

II. The left sum approximation of the total area between the curve and the x-axis from $x = -1$ to $x = 3$, using 4 equal subdivisions, is 6.

III. The left-sum approximation of $\int_{-1}^{3} f(t)\,dt$ with 4 equal subdivisions is 6.

A. I only B. II only C. II and III only D. I and II only E. I, II, and III

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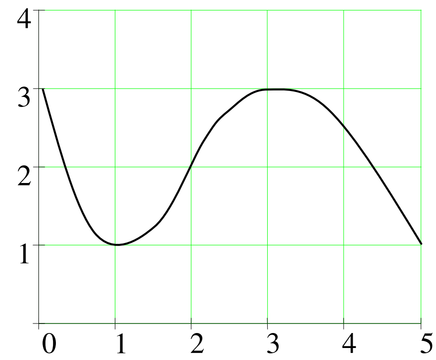
14. Use the Trapezoidal Rule with $n = 4$ to approximate the integral $\int_1^5 f(x)dx$ for the function f whose graph is shown at the right.



A. 8 B. 8.5 C. 9

D. 9.5 E. 10

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15. If $f(x)$ is a linear function with slope m, then $\int_a^b f'(x)dx$ has the value

A. 0 B. $b - a$ C. $m(b - a)$ D. $m(b^2 - a^2)$ E. $\frac{m}{2}(b^2 - a^2)$

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16. The approximate average rate of change of the function $f(x) = \int_0^x \sqrt{1 + \cos^2(t)}dt$ over the interval $[1, 5]$ is nearest to

A. 1.09 B. 1.12 C. 1.15 D. 1.18 E. 1.21

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17. $\int x\sqrt{3 + x}dx =$

A. $\frac{2}{5}(3 + x)^5 - 2(3 + x)^3 + C$

B. $\frac{2}{5}(3 + x)^{5/2} - 2(3 + x)^{3/2} + C$

C. $\frac{1}{2}(3 + x)^2 - \frac{3}{2}(3 + x) + C$

D. $\frac{1}{2}(3 + x)^4 - \frac{3}{2}(3 + x)^2 + C$

E. $\frac{1}{2}x^4 - \frac{3}{2}x^2 + C$

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18. The function defined on $[-1, 4]$ by $f(x) = \begin{cases} |x-2| & \text{for } 1 \le x \le 4 \\ -x & \text{for } -1 \le x < 1 \end{cases}$.

a. Sketch a graph of f.

b. If possible, determine values of x on $[-1, 4]$ for which f is *not* continuous. Explain briefly.

c. If possible, determine values of x on $[-1, 4]$ for which the derivative does not exist.

d. Evaluate: $\int_{-1}^{4} f(x)dx$.

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19. Consider the function $F(x) = kx^2 + 3$.

a. If the tangent lines to the graph of F at $(t, F(t))$ and $(-t, F(-t))$ are perpendicular, find t in terms of k.

b. Find the slopes of the tangent lines mentioned in part a.

c. Find the coordinates of the point of intersection of the tangent lines mentioned in part a.

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20. The temperature, in degrees Celsius $(°C)$, of the water in a pond is a differentiable function W of time t. The table at right shows the water temperature as recorded every 3 days over a 15-day period.

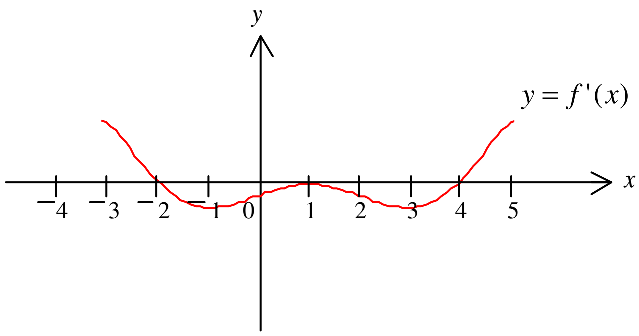
t (days)	$W(t)$ $(°C)$
0	20
3	31
6	28
9	24
12	22
15	21

a. Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

b. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

c. A student proposes the function P, given by $P(t) = 20 + 10te^{-t/3}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

d. Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.



21. The figure above shows the graph of f', the *derivative* of a function *f*. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

d. For what values of x does f have a relative maximum? Why?

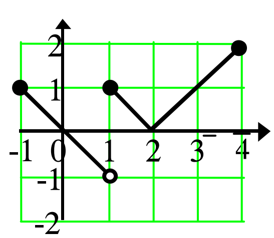
e. For what values of x does f have a relative minimum? Why?

f. On what intervals is the graph of f concave upward? Use f' to justify your answer.

g. Suppose that $f(1) = 0$. Draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

Answer

1. E
2. C
3. E
4. A
5. B
6. C
7. C
8. D
9. B
10. E
11. E
12. A
13. D
14. B
15. C
16. D
17. B
18. a.



b. $x = 1$

c. $x = 1, 2$

d. 2.5

19. a. $t = \frac{\pm 1}{2k}$

b. At $t = \frac{1}{2k}$, the slope is 1. At $-t = \frac{-1}{2k}$, the slope is -1.

c. $\left(0, \frac{-1}{4k} + 3\right)$

20. a. $\frac{-1}{2}$ degrees C per day

b. 25.1° C.

c. $P'(12) = -0.549$ ° C per day. The temperature of the water on the 12th day is dropping at a rate of 0.549° C per day.

d. 25.757° C.

21. a. $\{-2\}$ The derivative is positive to the left of -2 and negative to the right. OR The area under the curve changes from positive to negative at -2.

b. $\{4\}$ The derivative is negative to the right of 4 and positive to its right. OR The area under the curve changes from negative to positive at 4.

c. $\{-1 < x < 1 \cup 3 < x < 5\}$ The derivative is always increasing on a concave upward interval.

d. $0 < x < 2$