SUPPLEMENTAL PROBLEMS – THE INTEGRAL

Some problems on this sheet are taken from *Calculus; Single Variable*, second edition by Hughes-Hallett, Gleason, et al. New York, John Wiley & Sons, 1998.

1. A student is speeding down Route 252 in his fancy red Porche when his radar system warns him of an obstacle 400 feet ahead. He immediately applies the brakes, starts to slow down, and spots a skunk in the road directly ahead of him. He continues braking until he comes to a complete stop.

   Suppose that the "black box" in the Porche records the car's speed every two seconds, producing the following table. Assume that the speed decreases throughout the 10 seconds it takes to stop, although not necessarily at a uniform rate.

Time since brakes were applied (sec)	0	2	4	6	8	10
Velocity (ft/sec)	100	80	50	25	10	0

   a. Using the information in this table, what is your best estimate of the total distance that the student's car traveled before coming to rest?

   b. Which statement below can you justify from the information given in the story and data table?

      i. The car stopped before getting to the skunk.

      ii. The "black box" data is inconclusive. The skunk may or may not have been hit.

      iii. The unfortunate skunk was hit by the car.

2. Coal gas is produced at a gas works. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping in the gas.

Time (months)	0	1	2	3	4	5	6
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

   a. Make an overestimate and an underestimate of the total quantity of pollutants that escaped during the first month.

   b. Make an overestimate and an underestimate of the total quantity of pollutants that escaped during the first six months.

c. How often would measurements have to be made in order to find overestimates and underestimates which differ by less than 1 ton from the exact quantity of pollutants that escaped during the first six months?

3. An object starts at the origin and travels along the number line. Time, t, is given in seconds and the object's velocity, v, in meters/second, is given by $v(t) = 1 + t^2$ for $0 \le t \le 6$.

a. Use $\Delta t = 2$ to make a table for time and velocity

b. Find an over- and under-estimate for its position at time $t = 4$.

c. If the object had not started at the origin, but rather started at $x = 5$, where would it be at time $t = 4$? Give an over- and under-estimate.

d. Find an over- and under-estimate for the total distance traveled over the six seconds and then average the two.

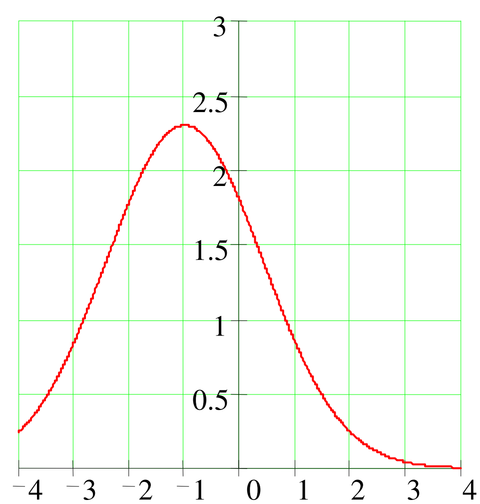
4. You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table below gives your acceleration, a (in m/sec^2), after t seconds.

t	0	1	2	3	4	5
a	9.81	8.03	6.53	5.38	4.41	3.61

a. Give upper and lower estimates of your speed at $t = 5$.

b. Get a new estimate by taking the average of your upper and lower estimates. What does the concavity of the graph of acceleration tell you about your new estimate?

5. Use the grid to find an *upper* and *lower* estimate of the area between the curve and the x-axis from $x = -3$ to $x = 3$.



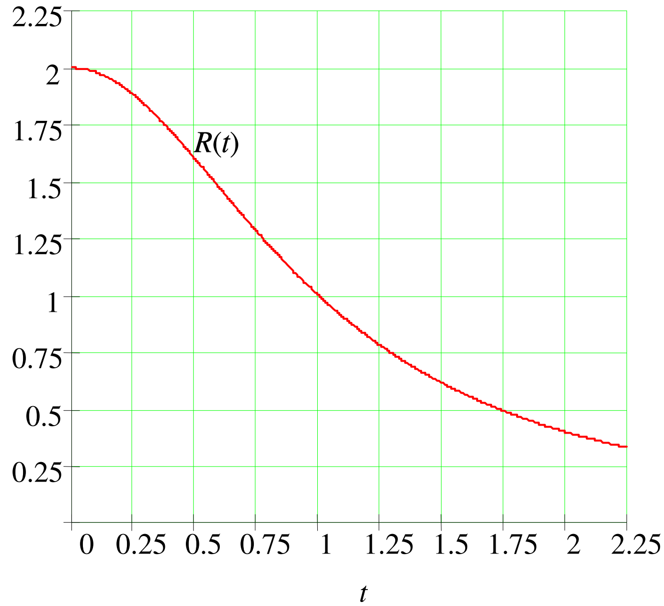
6. Find the area between the curve $y = x - x^2$ and the x-axis on the interval $[0, 4]$. Use the AREAPROX program to check your answer.

7. Water is leaking out of a tank at a rate of $R(t)$ gallons/hour, where t is measured in hours.

   a. Write a definite integral that expresses the total amount of water that leaks out in the first two hours.

   b. At right is a graph of $R(t)$. On the sketch, shade in the region whose area represents the total amount of water that leaks out in the first two hours.

   c. Give an upper and lower estimate of the total amount of water that leaks out in the first two hours.



8. A news broadcast in early 1993 said the average American's annual income is changing at a rate of $r(t) = 40(1.002)^t$ dollars per month, where t is in months, starting at January 1, 1993. Use the TI-89 to help you find the change in the average American's income during 1993?

9. A cup of coffee at 90°C is put into a 20°C room when $t = 0$. If the coffee's temperature is changing at a rate given in °C per minute by $r(t) = -7e^{-0.1t}$, t in minutes, find the coffee's temperature when $t = 10$. (Use TI-89.)

10. The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate (in billions of barrels per year) is given by the function $r = f(t)$, where t is measured in years and $t = 0$ is the start of 1990.

    a. Write a definite integral that represents the total quantity of oil used between the start of 1990 and the start of 1995.

    b. Suppose $r = 32e^{0.05t}$. Using a left-hand sum with five subdivisions, find an approximate value for the total quantity of oil used between the start of 1990 and the start of 1995.

    c. Interpret each of the five terms in the sum from part b in terms of oil consumption.

11. Make a table of values for the following function for $x = 0, 0.5, 1.0, 1.5, 2.0$: $A(x) = \int_0^x \sqrt{t^4 + 1}\,dt$
(Use the TI-89 integral function to evaluate the definite integral for each value.)

12. Use the TI-89 program ACCUMLAT to find values for the accumulation function and then plot them on a graph. Copy the graph onto paper and try to write a functional expression for the values plotted.

a. $f(t) = \dfrac{1}{2t}$ and $1 \le x \le 5$

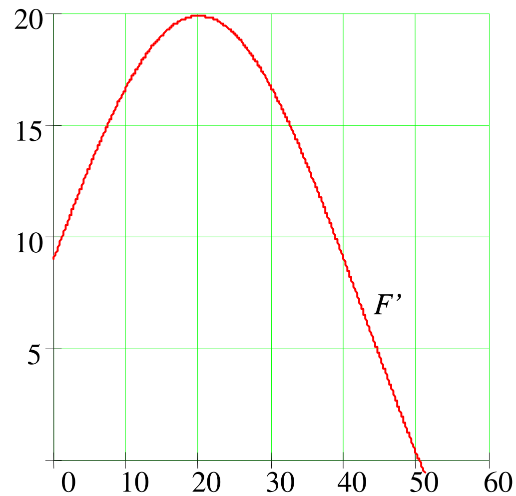
b. $f(t) = e^t$ and $0 \le x \le 5$

13. The graph of the derivative F' of some function F is given. Let $F(20) = 150$.

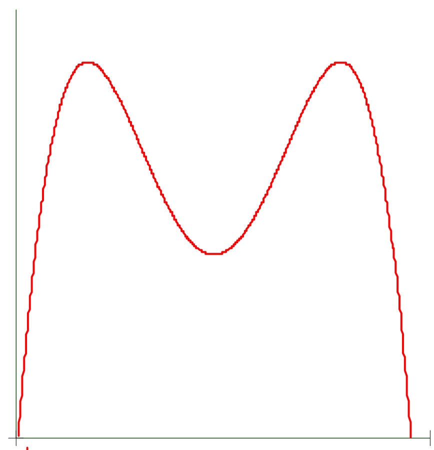
a. At what value for x will F attain its maximum.

b. Estimate the area under the curve of F' from $x = 20$ to $x = 50$.

c. Estimate the maximum value attained by F.



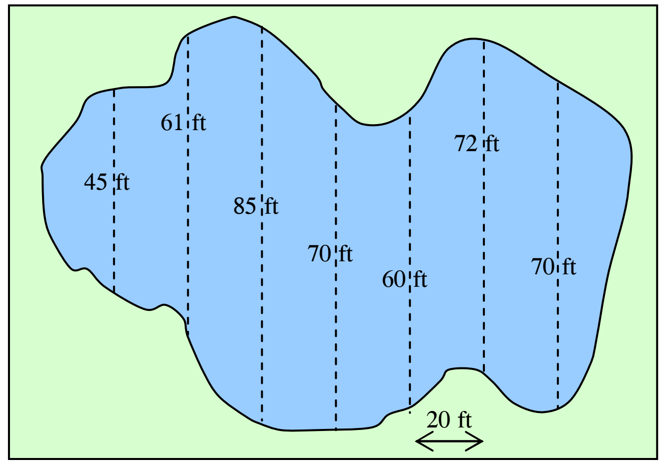
14. At right is a graph of $f(t)$. Draw a graph of $A(x)$ where $A(x) = \int_0^x f(t)\,dt$.



15. $\dfrac{d}{dx}\displaystyle\int_1^{2x} \cos(t^2)\,dt =$

16. $\dfrac{d}{dx}\displaystyle\int_1^{x^2} \dfrac{dt}{1+\sqrt{1-t}} =$

17. To estimate the surface of a pond, a surveyor takes several measurements, as shown in the figure. Estimate the surface area of the pond using the Trapezoidal Rule.



Answers

1. a. He traveled at most $100 \bullet 2 + 80 \bullet 2 + 50 \bullet 2 + 25 \bullet 2 + 10 \bullet 2 = 530$ feet.

   He traveled at least $80 \bullet 2 + 50 \bullet 2 + 25 \bullet 2 + 10 \bullet 2 + 0 \bullet 2 = 330$ feet.

   b. Inconclusive.

2. a. Overestimate is 7 tons; underestimate is 5 tons.

   b. Overestimate is 74 tons, underestimate is 59 tons.

   c. Every $\frac{1}{16}$ of a month, which is approximately 2 days.

3. a.

time	0	2	4	6
velocity	1	5	17	37

   b. 44; 12

   c. 49; 17

   d. 82 m

4. a. Overestimate is 34.16 m/s; underestimate is 27.96 m/s.

   b. Average is 31.06 m/s. New estimate corresponds to the area under the diagonal line connecting the starting point to the ending point. Since the curve is concave upward, this new estimate is higher than the actual value.

5. Upper estimate: 9.1

   Lower estimate: 5.4

7. a. $\int_0^2 R(t)dt$

   b. Upper estimate is 2.4 gallons; lower estimate is 2.0 gallons

8. $\int_0^{12} 40(1.002)^t \, dt = 485.80$. The average American's income increased by $485.80 in 1993.

9. $\int_0^{10} -7e^{-0.1t}\,dt = -44.2$. At $t = 10$ the temperature of the coffee is 45.8°C.

10. a. $\int_0^5 f(t)\,dt$

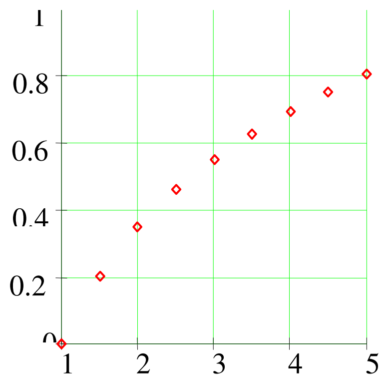
b. $32 \bullet 1 + 33.641 \bullet 1 + 35.365 \bullet 1 + 37.179 \bullet 1 + 39.085 \bullet 1 = 177.270$ barrels

c. Each term represents the oil consumption in that year.

11.

x	0	0.5	1	1.5	2
A	0	0.503	1.089	2.031	3.654

12. a.

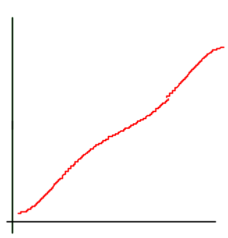


13. a. $x = 50$

b. 360

c. 510

14.



15. $2\cos(4x^2)$

16. $\dfrac{2x}{1+\sqrt{1-x^2}}$

17. 9260 ft^2