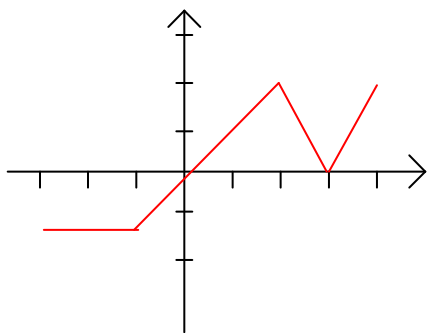


SUPPLEMENTAL PROBLEMS – DERIVATIVE APPLICATIONS

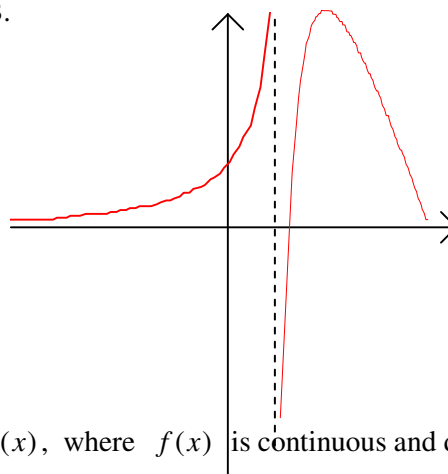
Some problems on this sheet are taken from *Calculus; Single Variable*, second edition by Hughes-Hallett, Gleason, et al. New York, John Wiley & Sons, 1998.

1. For each graph below, assume that it is a graph of $f'(x)$, where $f(x)$ is continuous and defined everywhere. Do the following for each:
 - a. List all the things you know about $f(x)$ by looking at the given graph.
 - b. Sketch a possible graph for $f(x)$.

A.

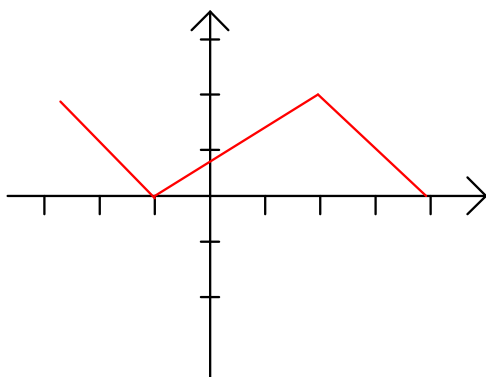


B.

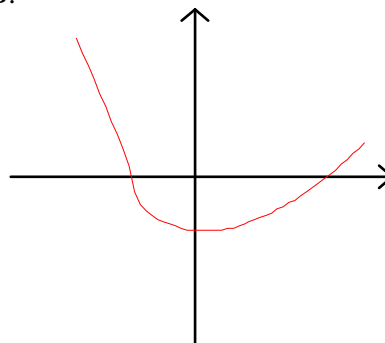


2. For each graph below, assume that it is a graph of $f''(x)$, where $f(x)$ is continuous and defined everywhere. Do the following for each:
 - a. List all the things you know about $f(x)$ by looking at the given graph.
 - b. Sketch a possible graph for $f(x)$.

A.



B.



3. Write the linear approximation to the curve $f(x) = \sqrt{x}$ at $x = 4$.

4. Given the following table of data:

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.7	3.5	3.5	3.9	4.0	3.9

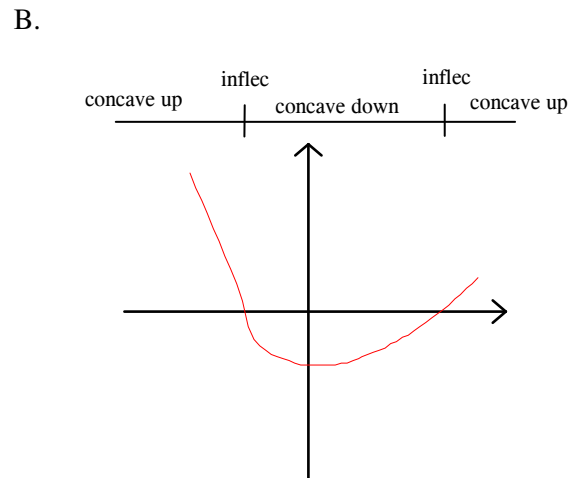
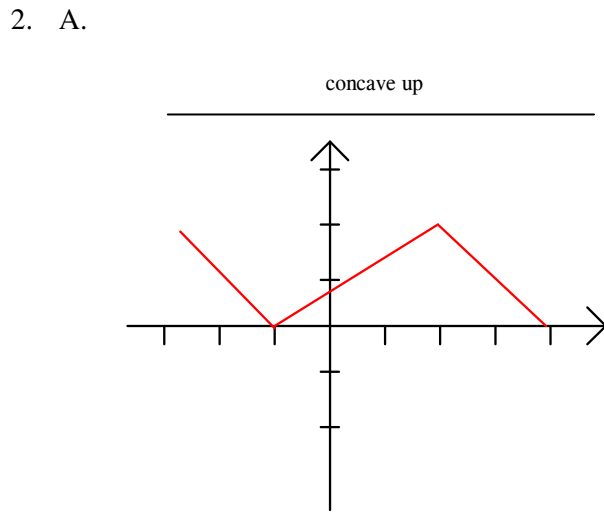
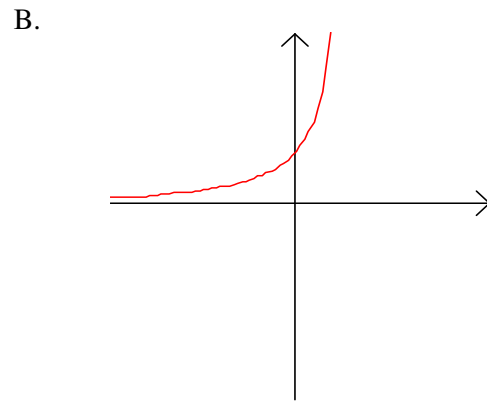
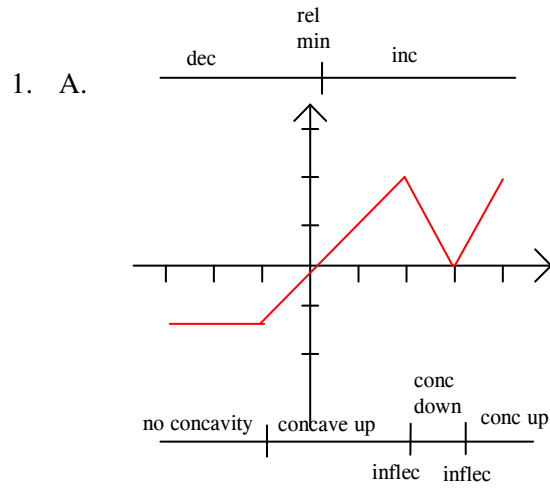
- Estimate an equation of the tangent line to the curve at $x = 0.6$.
- Using this equation, estimate $f(0.7)$, $f(1.2)$, and $f(1.4)$. Which of these estimates do you feel most confident about? Why?

5. Given the following table of data about a function:

x	6.5	7.0	7.5	8.0	8.5	9.0
$f(x)$	10.3	8.2	6.5	5.2	4.1	3.2

- Estimate $f'(7)$, $f'(8.5)$, and $f'(6.75)$.
 - Estimate the rate of change of f' at $x = 7$.
 - Find, approximately, a linear approximation to the curve of f at $x = 7$.
 - Estimate $f(6.8)$.
6. Find the linear approximation to the curve $y = \frac{1}{\sqrt{1+x}}$ at $x = 0$.
7. The cost in dollars of removing $p\%$ of the air pollutants in the stack emission of a utility company that burns coal to generate electricity is $C = \frac{80,000p}{100-p}$, where $0 \leq p < 100$. Use differentials to approximate the increase in cost if the government requires the utility company to remove 2% more of the pollutants and p is currently 40%.

Answers



3. $y \approx \frac{1}{4}(x-4)+2$

4. a. $y = 1.25(x-0.6)+3.9$

b. $f(0.7) \approx 4.025$, $f(1.2) \approx 4.65$, $f(1.4) \approx 4.9$ The first of these estimates is the only one that one would expect to be close. The other x values are too far away from the original point to be valid.

5. a. $f'(7) \approx -3.8$, $f'(8.5) \approx -2$, $f'(6.75) \approx -4.2$

b. The rate of change of f at $x = 7$ is approximately 1.6.

x	6.75	7	7.25
$f'(x)$	-4.2	-3.8	-3.4

c. $y \approx -3.8(x-7) + 8.2$

d. $f(6.8) \approx 8.96$

6. $y \approx \frac{-1}{2}(x) + 1$

7. \$4444.44