

## SUPPLEMENTAL PROBLEMS – THE DERIVATIVE

Some problems on this sheet are taken from *Calculus; Single Variable*, second edition by Hughes-Hallett, Gleason, et al. New York, John Wiley & Sons, 1998.

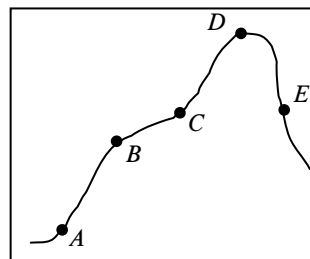
1. The table below gives the position  $s$  of car with respect to time  $t$ . Find the average velocity over each time interval. Use it to sketch a rough graph of the velocity with respect to time.

$t$ (sec)	0	0.2	0.4	0.6	0.8	1.0
$s$ (ft)	0	0.5	1.8	3.8	6.5	9.6

2. A particle is moving along a horizontal line so that its distance  $d$  from the origin depends on time  $t$  according to the equation:  $d = t^3 - 3t^2 + 1$ .
- Make a table of difference quotients from time  $t = 0$  to  $t = 4$ , using intervals for  $t$  of length 0.5.
  - Since a difference quotient tells us the average rate of change of  $d$  with respect to  $t$ , then the difference quotients should correspond to what aspect of this particle's motion?
  - Let motion to the right along the line be assigned positive values of velocity and motion to the left be assigned negative values of velocity. Looking at the table of difference quotients, tell over what approximate interval of time the particle is moving to the left.
  - Approximately when is the particle traveling at its slowest speeds?
  - Sketch a graph of the difference quotients, assigning the value to the midpoint of the interval.
  - Write an equation that reasonably well fits the graph of the difference quotients.
3. Describe the graph of a function near an  $x$ -value at which the difference quotients are:
- Zero
  - A maximum value.
4. Describe the graph of a function for which all of the difference quotients are:
- Zero.
  - Constant.
  - Changing, but always positive.
  - Changing, but always negative.

5. Refer to the graph at right.

- a. Between which pair of consecutive points is the average rate of change of the function greatest?
- b. Between which pair of consecutive point is the average rate of change of the function closest to zero?
- c. Between which two pairs of consecutive points are the average rates of change of the function closest?



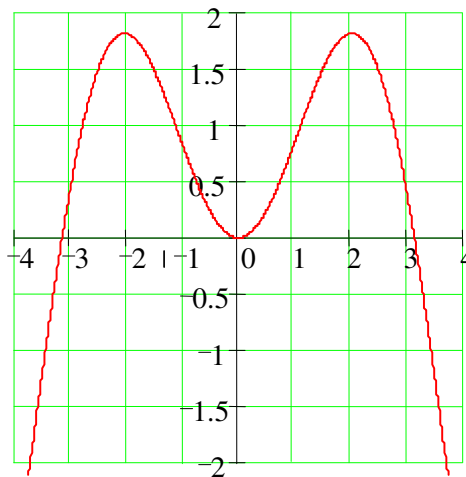
6. The population of deer is modeled by

$$P(t) = 4000 + 500 \sin\left(2\pi t - \frac{\pi}{2}\right) \text{ where } t \text{ is measured in years.}$$

- a. How does the population vary with time? Use your TI-89 to view a graph of  $P(t)$  for one year. Copy the graph onto your paper.
  - b. Use the graph to decide when in the year the population is a maximum. What is that maximum? Is there a minimum? If so, when?
  - c. Use the graph to decide when the population is growing fastest. When is it decreasing fastest?
  - d. Estimate roughly how fast the population is changing on the first of July.
7. A car is driven at high speed, and its speed then decreases slowly. Sketch a graph of the distance the car has traveled as a function of time.
8. A ball is tossed into the air from a bridge, and its height,  $f(t)$  (in feet), above the ground  $t$  seconds after it is thrown is given by  $f(t) = -16t^2 + 50t + 36$ .
- a. How high above the ground is the bridge?
  - b. What is the average velocity of the ball for the first second?
  - c. Approximate the velocity of the ball at  $t = 1$  second.
  - d. Graph the function  $f$ , and determine the maximum height the ball will reach. What should the velocity be at the time the ball is at the peak?
  - e. Use the graph to decide at what time,  $t$ , the ball reaches its maximum height.

9. Use your calculator to make a table of values, rounded to two decimal places, for  $f(x) = \log x$  (that is, log base 10) with  $x = 1, 1.5, 2, 2.5, 3$ . Then use this table to answer parts a. and b.

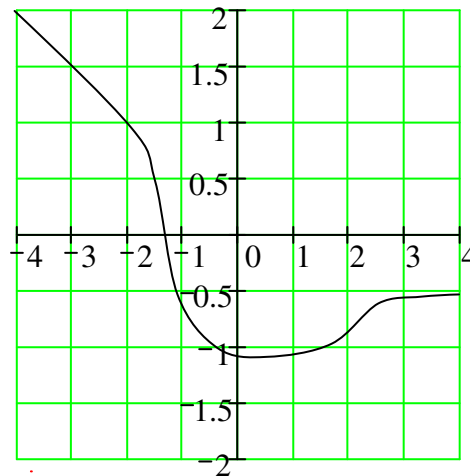
- a. Find the average rate of change of  $f(x)$  between  $x = 1$  and  $x = 3$ .
- b. Use average rates of change to approximate the instantaneous rate of change of  $f(x)$  at  $x = 2$ .



- 10. a. Use the graph at right to estimate the slope of the tangent line at every integer point on the curve. Write the values in the table below
- b. Plot the values of the derivative and sketch in its graph.

$x$	-3	-2	-1	0	1	2	3
$f'(x)$							

- 11. a. Use the graph at right to estimate the slope of the tangent line at every integer point on the curve. Write the values in the table below
- b. Plot the values of the derivative and sketch in its graph.



$x$	-3	-2	-1	0	1	2	3
$f'(x)$							

12. Sales  $f$ , in billions of dollars, of recreational vehicles in the United States for the years 1980 through 1994 are given by the function  $f(x) = 0.00077x^4 - 0.018x^3 + 0.08x^2 + 0.51x + 1.13$ , where  $x = 0$  in the year 1980.

- a. Graph the function on your TI-89. Use window settings of  $-1 \leq x \leq 15$  and  $0 \leq y \leq 6$  with a scale of 1. Then turn the grid on. To do this, *in the graphing window*, press F1, 9. Beside the word *Grid*, change the setting to ON.
- b. Make a hand drawn copy of the graph on graph paper.
- c. Use the graph to estimate the values of the derivative at each integer  $x$ -value and record in the table below.

$x$	0	1	2	3	4	5	6	7	8
$f'(x)$									

$x$	9	10	11	12	13	14
$f'(x)$						

- d. Plot the ordered pairs on the graph and make a sketch of the derivative.
- e. When is the instantaneous rate of change the greatest?
- f. Where is the instantaneous rate of change negative? What does this mean in terms of sales of recreational vehicles?
- g. Using this model, predict the number of sales in the year 2000? Does this seem reasonable? (Why, or why not?)

13. Given the numerical values shown, find approximate values for the derivative of  $f(x)$  at each of the  $x$  values.

$x$	0	1	2	3	4	5	6	7	8
$f(x)$	18	13	10	9	9	11	15	21	30
$f'(x)$									

- a. Where is the rate of change of  $f(x)$  positive?
- b. Where is it negative?
- c. Where does the rate of change of  $f(x)$  seem to be greatest?

14. Given the function  $f(x) = 3x^3 - 4x - 8$ .

- a. Find its derivative, assisted by your TI-89.
- b. Find the slope of the tangent line at  $x = 5$ .
- c. Where else does the curve have a tangent line with the same slope?
- d. Find the points on the curve where the tangent line is horizontal.

15. By zooming in on a TI-89 graph, determine if the function is locally linear at the designated point:

a.  $y = |x^2 - 4x - 12|$  at  $x = 6$

b.  $y = |x^3 - 2|$  at  $x = 0$

c.  $y = (x + 1)^{1/5}$  at  $x = -1$

d.  $y = \begin{cases} x^2 + 3 & x \leq 3 \\ 4x & x > 3 \end{cases}$

**For questions # 16 – 18, do each of the following steps:**

- a. Enter the function in your TI-89.
- b. Graph it in the *ZoomDecimal* Window.
- c. Copy the table below onto your paper.

$x$	-3	-2	-1	0	1	2	3
$f'(x)$							

- d. Use the TI-89 to draw a tangent line to the curve at each  $x$  value in the table above. Remember that the slope of the tangent line is the slope of the curve and the value of the derivative at that point. Use the equation of the tangent line (as displayed at the bottom of the graphing window) to fill in the table for the derivative value that point.
- e. Use the table to sketch a graph of the derivative.

16.  $y = 3x^3 - 5x$

17.  $y = \cos(x)$  Be sure calculator is in radian mode.

18.  $y = e^x$

**For questions # 19 – 21, do each of the following steps:**

- Enter the function into your TI-89.
- Graph it in the *ZoomDecimal* window.
- Zoom In* on the specified point several times.
- Use the concept of local linearity to answer the question, "Is the graph differentiable at the point?"
- If the graph is differentiable, estimate the derivative at the point by calculating the slope of the "line" that results from the zooming-in you did in part c. To calculate the slope, place the trace at two different points on the "line" and read off the coordinates of the point. Then use the usual algebra slope formula to calculate the slope of the line joining those two points.

19.  $y = \arctan x$  at  $\left(1, \frac{\pi}{4}\right)$  Be sure calculator is in radian mode.

20.  $y = 2x^{\frac{2}{3}}$  at  $(1, 2)$

21.  $y = \frac{1}{x}$  at  $\left(3, \frac{1}{3}\right)$

22. The temperature,  $T$ , in degrees Fahrenheit, of a cold yam placed in a hot oven is given by  $T = f(t)$ , where  $t$  is the time in minutes since the yam was put in the oven.

- What is the sign of  $f'(t)$ ? Why?
- What are the units of  $f'(20)$ ? What is the practical meaning of the statement  $f'(20) = 2$ ?

23. An economist is interested in how the price of a certain commodity affects its sales. Suppose that at a price of  $\$p$ , a quantity,  $q$ , of the commodity is sold. If  $q = f(p)$ , explain in economic terms the meaning of the statements  $f(10) = 240,000$  and  $f'(10) = -29,000$ .

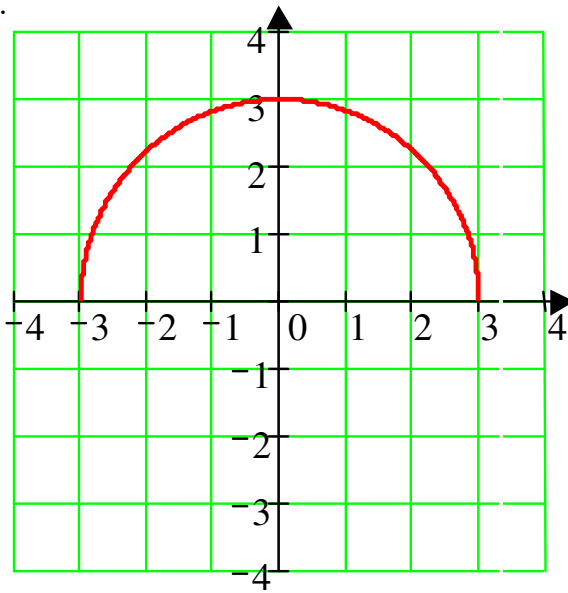
24. Suppose  $C(r)$  is the total cost of paying off a car loan borrowed at an annual interest rate of  $r\%$ . What are the units of  $C'(r)$ ? What is the practical meaning of  $C'(r)$ ? What is its sign?

25. An ice cream company knows that the cost,  $C$  (in dollars), to produce  $q$  quarts of cookie dough ice cream is an function of  $q$ , so  $C = f(q)$ . What are the units of  $f'(q)$ ? Explain the meaning of the statements  $f(200) = 70$  and  $f'(200) = 3$

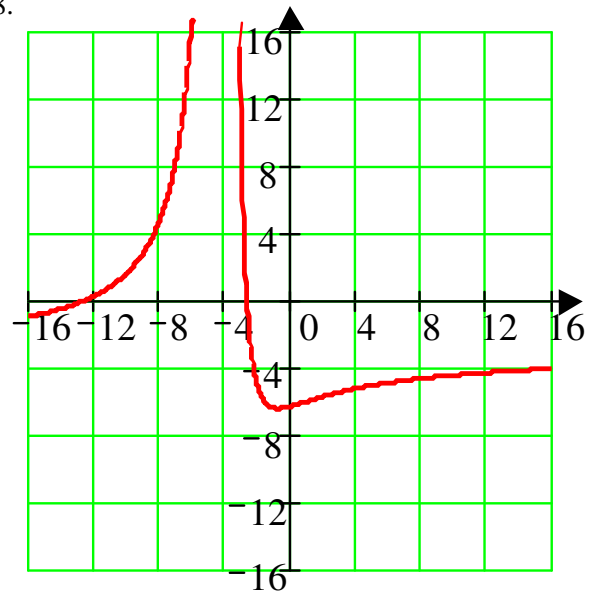
26. Write the linear approximation to the curve  $y = \sin\left(\frac{x}{2}\right)$  at  $x = \pi$

**Draw the derivative of each curve**

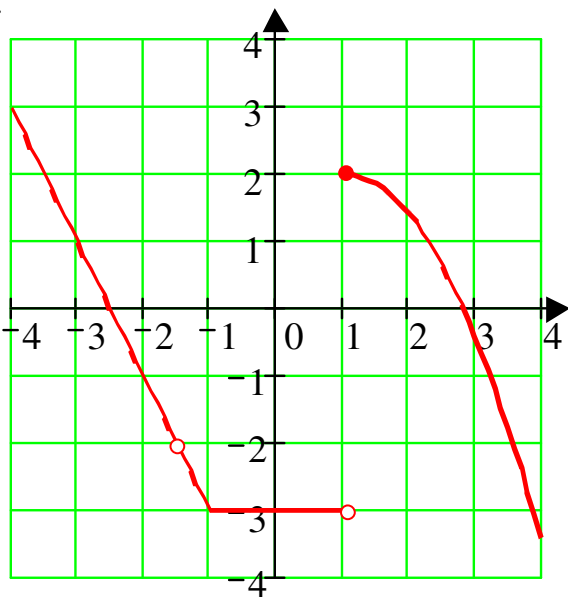
27.



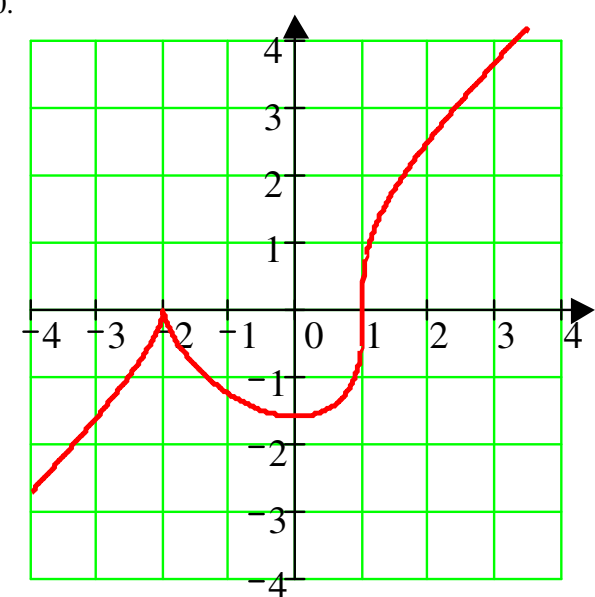
28.



29.



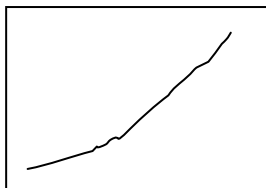
30.



Answers

1.

time	0 to 0.2	0.2 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1.0
Ave velocity	2.5	6.5	10	13.5	15.5



2. a.

time	0 to 0.5	0.5 to 1	1 to 1.5	1.5 to 2	2 to 2.5	2.5 to 3	3 to 3.5	3.5 to 4
Diff quot	-1.25	-2.75	-2.75	-1.25	1.75	6.25	12.25	19.75

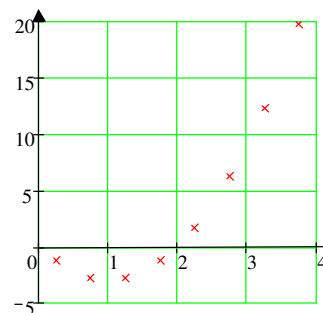
b. average velocity

c. (0, 2)

d. It starts slowly and must momentarily stop around 2 seconds to change directions.

f. perhaps quadratic,  $y = 3x^2 - 6x$

e.



3. a. flat, horizontal

b. steeply slanted

4. a. constant

b. linear

c. increasing

d. decreasing

5. a. between A and B.

b. between B and C.

c. A and B, C and D.

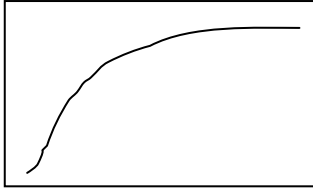
6. a. It varies periodically with a period of one year.

b. maximum on July 1, population of 4500, minimum on January 1, population 3500.

c. growing fastest around April 1, decreasing fastest around October 1.

d. rate of change is 0.

7.



8. a. 36 ft  
 b. 34 ft/sec  
 c. 18 ft/sec  
 d. approx 75 ft, velocity is 0  
 e. 1.563 sec.

9.

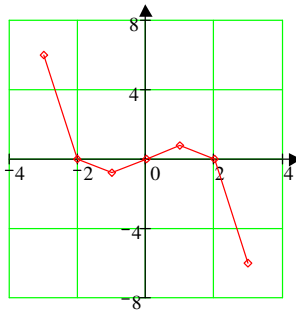
$x$	1	1.5	2	2.5	3
$\log(x)$	0	0.18	0.30	0.40	0.48

- a. 0.24  
 b. 0.22

10. a.

$x$	-3	-2	-1	0	1	2	3
slope	6	0	-0.8	0	0.8	0	-6

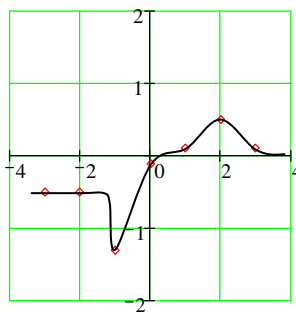
b.



11. a.

$x$	-3	-2	-1	0	1	2	3
slope	-0.5	-0.5	-1.3	-0.1	0.1	0.5	0.1

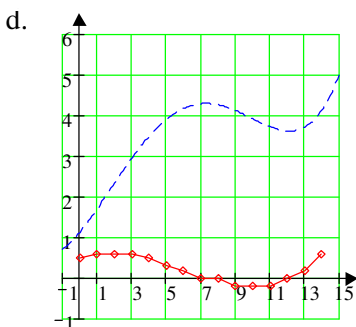
b.



12. c.

$x$	0	1	2	3	4	5	6	7	8
$f'(x)$	0.5	0.6	0.6	0.6	0.5	0.3	0.2	0	0

$x$	9	10	11	12	13	14
$f'(x)$	-0.2	-0.2	-0.2	0	0.2	0.6



- e. At approximately  $x=2$  or  $x=14$
- f. On the interval  $(8,12)$ . During this period of time, sales of recreational vehicles were decreasing.
- g. About 22.5 billion. This is probably an unreasonable expectation. The model is a quartic which will increase very rapidly in subsequent years.

13.

$x$	0	1	2	3	4	5	6	7	8
$f(x)$	18	13	10	9	9	11	15	21	30
$f'(x)$	-5	-4	-2	-0.5	1	3	5	7.5	9

- a. interval  $(4, 8)$
- b. interval  $(0, 3)$
- c.  $x=8$

14. a.  $f'(x) = 9x^2 - 4$

- b. 221
- c.  $x = -5$
- d.  $x = \pm \frac{2}{3}$

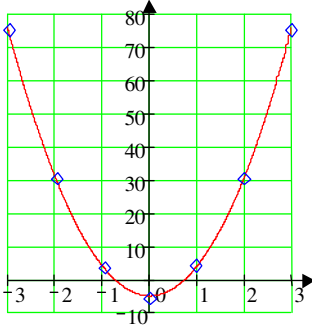
15. a. no

- b. yes
- c. yes
- d. no

16. d.

$x$	-3	-2	-1	0	1	2	3
$f'(x)$	76	31	4	-5	4	31	76

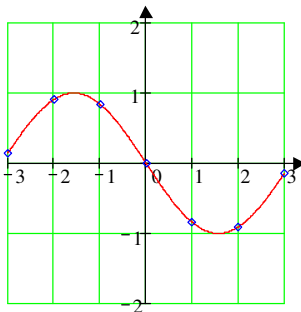
e.



17. d.

$x$	-3	-2	-1	0	1	2	3
$f'(x)$	0.14	0.91	0.84	0	-0.84	-0.91	-0.14

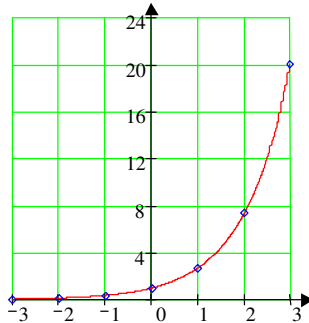
e.



18. d.

$x$	-3	-2	-1	0	1	2	3
$f'(x)$	0.05	0.14	0.37	1	2.72	7.39	20.09

e.



19. yes,  $\frac{1}{2}$

20. yes,  $\frac{4}{3}$

21. yes,  $-0.1$

22. a. The sign is positive because the temperature of the yam will be increasing.

b. The units are degrees Fahrenheit per minute.  $f'(20) = 2$  means that when the yam has been sitting in the oven for 20 minutes, its temperature will be changing at the rate of 2 degrees per minute.

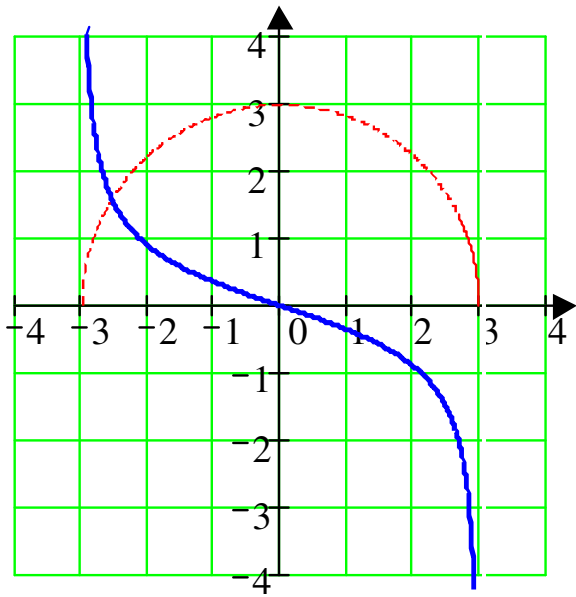
23.  $f(10) = 240,000$  means that if the price of each item is \$10, the company will sell 240,000 of the items.  $f'(10) = -29,000$  means that if the price is now \$10, then increasing the price by \$1 will cause the company to sell 29,000 FEWER of the items.

24. The units are dollars per percent.  $C'(r)$  is the number of dollars more that it will cost to pay off the loan when the interest rate is increased by 1% from its current rate of  $r$ . The sign of  $C'(r)$  should always be positive because a rising interest rate will always result in a greater cost to pay back the loan.

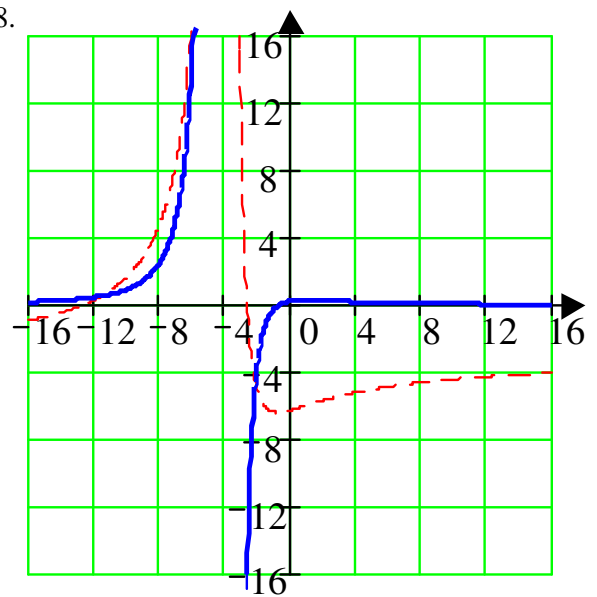
25. The units are dollars per quart.  $f(200) = 70$  means that it costs the company \$70 to produce 200 quarts of ice cream.  $f'(200) = 3$  means that if they are already producing 200 quarts of ice cream, it will cost them \$3 more to produce 1 more quart of ice cream.

26.  $y = 1$

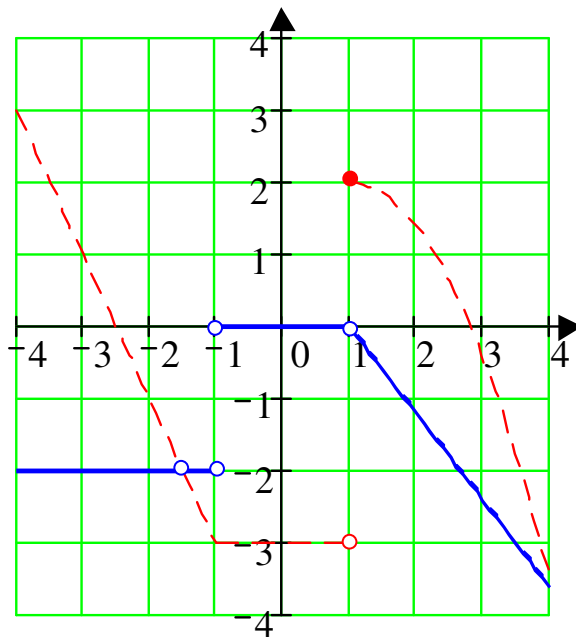
27.



28.



29.



30.

