Major Themes of Calculus Sure to Appear on the Semester Examination

Limits

- A limit is the single value to which an expression approaches as the x value approaches infinity or a real number.
- A limit can be evaluated by substitution if it yields a determinate form.
- If the limit is indeterminate upon substitution, the expression needs to be simplified using factoring or multiplying by the conjugate. If the expression contains an absolute value, it is often best to examine the left-hand and right-hand limits separately.
- Both left and right-hand limits must exist and be equal for the limit to exist at a point.

Continuity

- In general, continuity refers to the graph being "connected".
- To be continuous at a point: 1. the function must be *defined* at the point, 2. the *limit must exist* at the point, 3. the *value of the limit must be the defined value* of the function at that point.
- When checking to see if a piecewise function is continuous: 1. check that each separate piece is continuous over its restricted domain, 2. check that the function is defined at the connecting point between the two branches, 3. check that the limit of the left branch of the curve equals the limit of the right branch of the curve

Difference Quotients

- A difference quotient on a function y is the change in y over the change in x.
- A difference quotient gives the slope between *two* fixed points on a graph.
- The difference quotient on a function y gives the *average* rate of change of y with respect to x.
- If y represents a position or distance and x represents time, then the difference quotient gives the *average* velocity.

Derivatives

- The derivative is the *limit* of a different quotient as the change in x approaches zero.
- The derivative of a function gives the slope of a curve at a *single* fixed point.
- The derivative of y gives the *instantaneous* rate of change of y with respect to x.

- If y represents a position or distance, then the derivative of y gives the *instantaneous* velocity.

- If y represents a velocity function, then the derivative of y gives the *instantaneous* acceleration.

- A derivative cannot exist where a function is undefined, discontinuous, has a corner on its graph, or has a vertical tangent line.

- To determine if a piecewise defined function is differentiable: 1. check that each branch is differentiable over its restricted domain, 2. check that the function is *continuous* at the connecting point, 3. check that the *limit of the derivative* of the left branch equals the *limit of the derivative* of the right branch at the connecting point.

- All differentiable curves are continuous.

- If the derivatives of two different functions are equal, then the functions differ only by a constant.

Tangent Lines

- The slope of the tangent line at a point is the value of the derivative at that point.

- A tangent line is used as a linear approximation of a curve near the point of tangency.

- The differential uses the tangent line to approximate the change in y for a given change in x.

Relating Derivatives to Graphs

- If the derivative of a function is positive, the function is increasing; if the derivative of a function is negative, the function is decreasing.

- To find critical points, find all points (in the domain of the function) where the derivative is zero or undefined.

- If a function has a critical point where the derivative changes from positive to negative, the point is a relative maximum. If a function has a critical point where the derivative is zero and the function is concave downward, the point is a relative maximum.

- If a function has a critical point where the derivative changes from negative to positive, the point is a relative minimum. If a function has a critical point where the derivative is zero and the function is concave upward, the point is a relative minimum.

- To find an *absolute* maximum or minimum, find all relative extrema and compare their y-values with any endpoints on the function. If the function does not have endpoints, do one of the following: 1. compare the relative extrema with the limit of the function as x approaches positive infinity and as x approaches negative infinity, or 2. make an argument based on where the function is increasing or decreasing.

- To find points of inflection, find all points (in the domain of the function) where the second derivative is zero or undefined *and* where the second derivative changes sign on either side of the point.

- If the second derivative is positive, the graph is concave upward. If the second derivative is negative, the graph is concave downward. Visually, if the tangent line is below the graph, the graph is concave upward. If the tangent line is above the graph, the graph is concave downward.

Definite Integrals

- The average value of a function on an interval is the definite integral of the function divided by the width of the interval.

- The average value of a function corresponds to the height of a rectangle whose area is the same as the area under the curve.

- The definite integral is the limit of a summation of terms, called a Riemann Sum. Each term is the product of the y value of the function times a small interval of change in x. The limit is taken as the interval of change approaches zero.

- The First Fundamental Theorem of Calculus tells us that the definite integral can be evaluated by an entirely different method, using the antiderivative of the function.

- The Second Fundamental Theorem of Calculus tells us that if we let the upper bound on the definite integral of f be a variable, the new function generated will be an antiderivative of f. We call this antiderivative an accumulation function because it accumulates positive and negative area under the curve as x moves along the axis.

- The definite integral gives the area under a curve, provided the curve is always *above* the x-axis. If the function is sometimes below the x-axis, its area is the definite integral of the absolute value of the function.

- The definite integral of a *rate of change function, meaning* f', gives the *total change* in the original function f. If you need the *total amount*, you need to add this change to the starting amount.

- The definite integral of a velocity function gives the total *change in position* of the object. It DOES NOT give the total *distance* traveled unless the velocity has all been in one direction (that is, all positive or all negative) If the velocity changes direction, you get the total distance by integrating the absolute value of the velocity function, or by splitting the integral up into pieces over which the velocity function does not change sign.

- All continuous curves are integrable.

Indefinite Integrals

- The indefinite integral represents the family of all antiderivatives of a function. Remember that the family of antiderivatives is expressed with a $+C$ on the end.

- Techniques for finding an indefinite integral:

  a. Observe a power function structure (with the appropriate factor necessary for the chain rule).

  b. Split it into separate terms.