

Limits of the Form $\frac{0}{0}$

In this laboratory assignment you will study $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ where both $f(x)$ and $g(x)$ approach zero as x approaches a . This is known as a limit of the form $\frac{0}{0}$. Limits of this form are included in a larger class of limits known as indeterminate forms. The word indeterminate does not necessarily mean that the limit does not exist. It simply means that evaluating the limit in its present form is not possible.

In the past we have learned several techniques for evaluating such limits. If the expression is rational, we try to factor the numerator and denominator and reduce. We proved by using trigonometric relationships that the limit of $\frac{\sin x}{x}$ as x approaches 0 is 1. Each of these techniques works only on a specific type of expression. What we are looking for in this laboratory exercise is a more general approach to such limits that will work on a wide class of functions.

Listed below are the pairs of functions which you are to study on StudyWorks:

$$f(x) = \sin(2x) \text{ and } g(x) = x$$

$$f(x) = \sin(3x) \text{ and } g(x) = 5x$$

$$f(x) = \cos(x) - 1 \text{ and } g(x) = x$$

$$f(x) = e^x - 1 \text{ and } g(x) = x$$

$$f(x) = \ln(x+1) \text{ and } g(x) = x^2 + 2x$$

$$f(x) = \ln(x+1) \text{ and } g(x) = x^2 + 7x$$

$$f(x) = \ln(x+1) \text{ and } g(x) = \log(x+1)$$

$$f(x) = \sin(x) \text{ and } g(x) = x^2$$

1. Load the StudyWorks template from the file C-LAB-4.MCD. (My Computer Local Disk (c:) Studywks Ahlborn-Studyworks C-LAB-4.MCD)

2. For each pair of functions above:
 - a. Type in the two functions.
 - b. Use StudyWorks to construct a table of values for x , $f(x)$, $g(x)$, and $\frac{f(x)}{g(x)}$ for positive values of x close to zero. The StudyWorks worksheet is set up to accomplish this by generating a sequence of x -values that starts with 0.5 and repeatedly halves x until reaching a value close to zero.
 - c. Look at the graph of $f(x)$ and $g(x)$ as x nears zero. What information about the ratio $\frac{f(x)}{g(x)}$ can be obtained from the graph? How are the rates of change (slopes of tangents) of f and g near $x = 0$ related to the value of $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$? (Think about the concept of local linearity.)
 - d. Find a value for $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$.
 - e. Describe how the derivatives of f and g are related to the graphs of f and g near $x = 0$. How do these derivatives relate to the value of the limit you are studying?
3. Prepare a written report which includes each of the following:
 - a. This assignment paper as a cover to the report. Please fill in your name(s) and the due date. If you had a partner but are turning in separate reports, put the partner's name in parentheses.
 - b. An introductory paragraph about the nature and purpose of the assignment.
 - c. Answer to question 2.d. for each function.
 - d. A summary of your results, interpretations, and conclusions, paying particular attention to the questions raised in parts 2c. and 2e.
 - e. A print-out of a completed Studyworks template from any four of the functions you choose.