INTEGRATION TECHNIQUES AND APPLICATIONS

1. Def. **Smooth Curve**: A curve defined by $y = f(x)$ on the interval $[a, b]$ is called smooth if $f'(x)$ is continuous on $[a, b]$.

2. Def. **Arc Length**: Let $y = f(x)$ be a smooth curve on the interval [*a, b*] then the arc length of f between a and b is given by $s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$. Similarly, for a smooth curve given by $x = g(y)$, the arc length of g between c and d is $s = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$.

3. Def. **Area of a Surface of Revolution**: If $y = f(x)$ is a smooth curve on the interval [*a, b*], then the area of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is $2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx$ where $r(x)$ is the distance between the graph of f and the axis of revolution.

4. Def. **Pressure**: Pressure is the force *per* unit area on the surface of a body.

5. Thm. **Fluid Pressure**: Fluid Pressure P on an object is the density D of the fluid times the depth h of the object in the liquid. $P = Dh$.

6. Thm. **Fluid Force**: The fluid force FF on a submerged *horizontal* surface of area A is the pressure on the surface times the area of the surface. $FF = PA$.

7. Thm. **Integration by Parts**: $\int u\,dv = uv - \int v\,du$.

8. Thm. **Trigonometric Substitutions:** To integrate a radical expression of the form:

   a. $\sqrt{a^2 - x^2}$, substitute $x = a\sin\theta$.

   b. $\sqrt{x^2 + a^2}$, substitute $x = a\tan\theta$.

   c. $\sqrt{x^2 - a^2}$, substitute $x = a\sec\theta$.

9. Thm. **Partial Fractions with Non-Repeated Linear Factors**: A fraction of the form $\dfrac{P(x)}{(ax+b)(cx+d)}$ where $P(x)$ is a polynomial of degree less than the denominator can always be rewritten in the form $\dfrac{P(x)}{(ax+b)(cx+d)} = \dfrac{A}{(ax+b)} + \dfrac{B}{(cx+d)}$ where A and B are constants. This statement can be generalized for a fraction with n linear factors in the denominator.

10. Thm. **Partial Fractions with Repeated Linear Factors:** A fraction of the form $\frac{P(x)}{(ax+b)^3}$ where $P(x)$ is a polynomial of degree less than the denominator can always be rewritten in the form $\frac{P(x)}{(ax+b)^3} = \frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}$ where A, B, and C are fractions. This statement can be generalized for a fraction with n repeated linear factors in the denominator.

11. Thm. **Partial Fractions with a Non-Repeated Quadratic Factor**: A fraction of the form $\frac{P(x)}{(ax+b)(cx^2+dx+e)}$ where $P(x)$ is a polynomial of degree less than the denominator can always be rewritten in the form $\frac{P(x)}{(ax+b)(cx^2+dx+e)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(cx^2+dx+e)}$ where A, B, and C are constants.

----------*End for Chapter 7 Test*----------