THE INTEGRAL

1. Def. **Area Existence**: If f is a continuous function on $[a, b]$, then the limits as $n \to \infty$ of both the lower and upper sums exist and are equal to each other.

2. Def. **Area Under a Curve**: Let f be a continuous function on the interval $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is:

$$area = \lim_{n\to\infty}\left[\sum_{i=1}^{n} \left|f(c_i)\right| \bullet \Delta x\right] \text{ where } \Delta x = \frac{b-a}{n} \text{ and } x_{i-1} \le c_i \le x_i .$$

3. Thm. **Properties of Summations**:

a. $\displaystyle\sum_{i=1}^{n} c = cn$

b. $\displaystyle\sum_{i=1}^{n} c \bullet f(i) = c \bullet \sum_{i=1}^{n} f(i)$

c. $\displaystyle\sum_{i=1}^{n} \left[f(i) \pm g(i)\right] = \sum_{i=1}^{n} f(i) \pm \sum_{i=1}^{n} g(i)$

---*End for Chapter 4 Quiz*---

4. Def. **Riemann Sum**: Let f be defined on the closed interval $[a, b]$ which is partitioned by the set $\{a = x_0, x_1, x_2, \cdots, x_n = b\}$. If $c_i \in [x_{i-1}, x_i]$ and $\Delta x_i = x_i - x_{i-1}$, then the sum $\displaystyle\sum_{i=1}^{n} f(c_i)\Delta x_i$ is called a Riemann sum of f for the given partition.

5. Def. **Norm of the Partition**: The length of the largest subinterval of a partition is called the norm of the partition and is denoted by $\|\Delta\|$.

6. Def. **Regular Partition**: If every subinterval in a partition is of equal length, the partition is called regular.

*7. Def. **Definite Integral**: If f is defined on the interval $[a, b]$ and the limit of the Riemann sum $\displaystyle\lim_{\|\Delta\|\to 0}\sum_{i=1}^{n} f(c_i)\Delta x_i$ exists, then this limit is called the definite integral of f on $[a, b]$ and is denoted by $\displaystyle\int_a^b f(x)dx = \lim_{\|\Delta\|\to 0}\sum_{i=1}^{n} f(c_i)\Delta x_i$. (The values a and b are called the lower and upper limits of the integral, respectively.)

8. Def. **Integrable Function**: A function is said to be integrable on an interval if it has a definite integral on the interval.

9. Thm. **Continuity and Integrability**: If a function f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

10. Thm. **Area Under a Curve**: Let f be a continuous, function on the interval $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is found by:

$$area = \int_a^b |f(x)|dx .$$

11. Def. **Definite Integral at a Point**: $\int_a^a f(x)dx = 0$.

12. Def. **Definite Integral over a Reverse Interval**: If f is an integrable function on $[a, b]$, then $\int_b^a f(x)dx = -\int_a^b f(x)dx$.

13. Thm. **Definite Integral over Two Adjacent Intervals**: If f is an integrable function on the three intervals indicated, then $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

14. Thm. **Definite Integral of a Constant Times a Function**: If f is an integrable function on $[a, b]$, then $\int_a^b k \bullet f(x)dx = k \bullet \int_a^b f(x)dx$.

15. Thm. **Definite Integral of a Sum or Difference of Functions**: If f and g are both integrable functions on $[a, b]$, then $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.

16. Thm. **Comparing Definite Integrals**: If f and g are both integrable functions on $[a, b]$ and $f(x) \le g(x)$ for $a \le x \le b$, then $\int_a^b f(x)dx \le \int_a^b g(x)dx$.

17. Thm. **Integration of an Odd Function**: If f is an odd function which is integrable on $[-a, a]$, then $\int_{-a}^a f(x)dx = 0$.

18. Thm. **Integration of an Even Function**: If f is an even function which is integrable on $[-a, a]$, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$.

19. Thm. **Distance versus Displacement**: If the continuous function *v(t)* represents the velocity of a function over an interval of time [*a, b*], then:

    a. Displacement (or change in position) = $\int_a^b v(t)dt$.

    b. Total Distance Traveled = $\int_a^b |v(t)|dt$

*20. Thm. **Fundamental Theorem of Calculus**: If a function f is continuous on the closed interval [*a, b*] and F is an antiderivative of f on the interval [*a, b*], then $\int_a^b f(x)dx = F(b) - F(a)$.

Restatement: The total change in a function is the definite integral of its rate of change.

21. Def. **Antiderivative**: If *f′(x)* is the derivative of *f(x)*, then *f(x)* is called the antiderivative of *f′(x)*.

22. Thm. **Relationship Between Antiderivatives**: If F and G are both antiderivatives of a function *f*, then $F(x) = G(x) + C$ for some constant *C*.

23. Def. **Average Value of a Function**: If f is a continuous function on [*a, b*], then the average value of f on [*a, b*] is found by $\bar{y} = \frac{1}{b-a}\int_a^b f(x)dx$.

*24. Thm. **Average Value Theorem for Integrals**: If $f(x)$ is continuous on [*a, b*], then there exists a number c in (*a, b*) such that $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$. (Note that in the Larson text this theorem is referred to as the Mean Value Theorem for Integrals.)

Restatement: Under the given conditions, there is a point in the open interval where the value of the function is equal to the average value of the function over the interval.

Geometric Interpretation: Under the given conditions, there is a point in the open interval where the value of the function corresponds to the height of a rectangle, with base $(b-a)$, whose area is the same as the area under the curve between the two endpoints.

25. Def. **Accumulation Function**: Let f be continuous on [*a, b*] and $x \in [a,b]$. The function $A(x) = \int_a^x f(t)dt$ is called the accumulation function.

*26. Thm. **Second Fundamental Theorem of Calculus**: If f is continuous on an open interval containing a, then for every x in the interval where $F(x) = \int_a^x f(t)dt$, *F(x)* is an antiderivative of *f(x)*.

*27. Def. **Indefinite Integral**: The expression $\int f(x)dx$ is called the indefinite integral and is used to represent the family of all antiderivatives of $f(x)$.

28. Thm. **Properties of the Indefinite Integral**:

a. $\int dx = x + C$.

b. $\int k \bullet f(x)dx = k \bullet \int f(x)dx$.

c. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$.

--*End for Chapter 4 Test*--

29. Thm. **Trapezoidal Rule**: If f is continuous on $[a, b]$, then
$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)\right].$$

30. Thm. **Simpson's Rule**: If f is continuous on $[a, b]$ and T represents the approximation of $\int_a^b f(x)dx$ using the Trapezoidal Rule and M represents the same integral approximated with the same number of subdivisions using the Midpoint Rule, then the value $S = \frac{2M + T}{3}$ is the Simpson's Rule approximation for the integral.