DERIVATIVE APPLICATIONS

1. Def. **Absolute (or Global) Maximum**: $f(c)$ is a absolute (or global) maximum of f if c is in the domain of f and $f(c) \geq f(x)$ for every x in the domain of *f*.

2. Def. **Absolute (or Global) Minimum**: $f(c)$ is a absolute (or global) minimum of f if c is in the domain of f and $f(c) \leq f(x)$ for every x in the domain of *f*.

*3. Thm. **Extreme Value Theorem**: If $f(x)$ is continuous on $[a, b]$, then f has both a maximum and minimum value on the interval.

4. Def. **Relative (or Local) Maximum**: If there is an open interval containing c for which $f(c) \geq f(x)$ for all x in the interval, then $f(c)$ is called a relative (or local) maximum of *f*.

5. Def. **Relative (or Local) Minimum**: If there is an open interval containing c for which $f(c) \leq f(x)$ for all x in the interval, then $f(c)$ is called a relative (or local) minimum of *f*.

*6. Def. **Critical Number**: If f is defined at *c*, and $f'(c) = 0$ or $f'(c)$ is undefined, then c is called a critical number of *f*.

7. Thm. **Finding Relative Extrema**: If f has a relative maximum or minimum at $x = c$, then c is a critical number of *f*.

*8. Thm. **Rolle's Theorem**: If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

*9. Thm. **Mean Value Theorem**: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Geometric Interpretation: Under the given conditions, there is a point in the open interval where the tangent to the curve is the same as the slope of the line joining the endpoints.

Application: Under the given conditions, there is a point in the open interval where the instantaneous rate of change is the same as the average rate of change on the interval. If the function is a position function, then there is a point in the open interval where the instantaneous velocity is the same as the average velocity on the interval.

10. Def. **Increasing Function**: A function $f(x)$ is increasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

11. Def. **Decreasing Function**: A function $f(x)$ is decreasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

12. Def. **Strictly Monotonic**: A function is called strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

13. Thm. **Interpreting the Derivative**: Let $f(x)$ be a function that is differentiable on the open interval (a, b). Then:

   a. If $f'(x) > 0$ for all x in (a, b), then $f(x)$ is increasing on (a, b).

   b. If $f'(x) < 0$ for all x in (a, b), then $f(x)$ is decreasing on (a, b).

   c. If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant on (a, b).

14. Thm. **First Derivative Test for Local Extrema**: Let c be a critical number of the function f that is continuous on an open interval. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

   a. If f' changes from negative to positive at c, then f has a relative minimum at c.

   b. If f' changes from positive to negative at c, then f has a relative maximum at c.

   c. If f' does not change signs at c, then f has neither a relative maximum nor a relative minimum at c.

--*End for Chapter 3 Quiz*--

15. Def. **Concave Upward**: The graph of a differentiable function f is concave upward on an interval if f' is increasing on the interval.

16. Def. **Concave Downward**: The graph of a differentiable function f is concave downward on an interval is f' is decreasing on the interval.

17. Thm. **Test for Concavity**: Let $f(x)$ be a function whose second derivative exists on an open interval (a, b). Then:

   a. If $f''(x) > 0$ for all x in (a, b), then the graph of $f(x)$ is concave upward.

   b. If $f''(x) < 0$ for all x in (a, b), then the graph of $f(x)$ is concave downward.

*18. Def. **Inflection Point**: If the graph of f changes concavity at c and there exists a tangent line to the curve at c, then c is called an inflection point of f.

19. Thm. **Finding Inflection Points with the Second Derivative**: If c is a point of inflection of the graph of f, then either $f''(c) = 0$ or f'' is undefined at $x = c$.

20. Thm. **Second Derivative Test for Relative Extrema**: Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on some open interval containing c.

   a. If $f''(c) > 0$, then $f(c)$ is a relative minimum.

   b. If $f''(c) < 0$, then $f(c)$ is a relative maximum.

   c. If $f''(c) = 0$ or does not exist, the test fails.

--*End for Chapter 3 Test*---

*23. Def. **Differential of *y***: Let $y = f(x)$ represent a differentiable function and let dx be any nonzero change in x. Then the differential of y, written dy is given by $dy = f'(x)dx$.

24. Def. **Linear Approximation of** Δy : dy is called the linear approximation of the actual increment, Δy.

25. Def. **Linear Approximation of *f(x)***: The expression $f(x) \approx f'(a)(x-a) + f(a)$ is called the linear approximation to *f(x)* near $x = a$.

26. Def. **Cost Function**: The cost function *C(x)* represents the total cost of producing x number of units of some item.

27. Def. **Revenue Function**: The revenue function *R(x)* represents the total money taken in when selling x number of units of some item.

28. Def. **Profit Function**: The profit function *P(x)* is the difference between the revenue function and the cost function.

29. Def. **Marginal Cost**: The derivative of the cost function is called the marginal cost and represents the additional cost of producing 1 more item.

30. Def. **Marginal Revenue**: The derivative of the revenue function is called the marginal revenue and represents the additional money taken in upon selling 1 more item.