THE DERIVATIVE

**** Indicates that the item should be memorized in exact detail. You may be asked to quote it on a quiz or test.***

1. Def. **Average Velocity**: The average velocity of an object over an interval of time is the net change in position during the interval divided by the change in time. For a function *s(t)*, that is $\bar{v} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$.

2. Def. **Instantaneous Velocity**: The instantaneous velocity of an object at time t_1 is given by the limit of the average velocity as t_2 approaches t_1. For the function *s(t),* that is $v(t_1) = \lim\limits_{t_2 \to t_1} \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$ (provided the limit exists).

*3. Def. **Difference Quotient**: The expression $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$ is called a difference quotient and represents the average rate of change of *f(x)* over the interval $[x_1, x_2]$.

Alternate notation for a difference quotient is $\dfrac{f(x+h) - f(x)}{h}$ or $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$.

*4. Def. **Derivative**: $f'(x_1) = \lim\limits_{x_2 \to x_1} \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$ (provided the limit exists) is called the derivative of *f(x)* at x_1 and represents the instantaneous rate of change of *f(x)* at the point x_1.

*5. Def. **Derivative, Alternate Forms**: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ (provided the limit exists) or $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ (provided the limit exists).

6. Def. **Tangent Line**: If $f(x)$ is defined on an open interval containing c, and if the derivative $f'(c)$ exists, then the line passing through $(c, f(c))$ with slope $f'(c)$ is the tangent line to the graph of $f(x)$ at the point $(c, f(c))$.

----------*End for Chapter 1 Test*----------

8. Def. **Local Linearity**: A curve is called locally linear over an interval when zooming in on the curve causes it to look like a straight line.

9. Def. **Slope of a Curve**: The slope of a curve at a point is the slope of the tangent line at the point.

7. Def. **Normal Line**: A normal line to a curve at a point is a line perpendicular to the tangent line at the point.

10. Def. **Differentiability**: A function is said to be differentiable at a point if it has a derivative at the point.

11. Def. **Vertical Tangent Line**: If $f(x)$ is continuous at $x=c$ and $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=+\infty$ (or $-\infty$), then the line $x=c$ is called the vertical tangent to the curve at $(c, f(c))$.

12. Thm. **Local Linearity and Differentiability**: If a curve is locally linear at a point $x=c$ and the tangent line is not vertical there, then the function is differentiable at $x=c$.

13. Thm. **Continuity and Differentiability**: If a curve is differentiable at a point $x=c$, then it is continuous at $x=c$.

14. Def. **Right-Hand Derivative**: The right-hand derivative of $f(x)$ is $\lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}$, provided the limit exists.

15. Def. **Left-Hand Derivative**: The left-hand derivative of $f(x)$ is $\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h}$, provided the limit exists.

16. Thm. **Properties of Derivatives**:

    a. If $y=c$, then $y'=0$.

    b. If $y=c\bullet f(x)$, then $y'=c\bullet f'(x)$.

    c. If $y=f(x)\pm g(x)$, then $y'=f'(x)\pm g'(x)$.

    d. If $y=x^n$, then $y'=nx^{n-1}$.

17. Thm. **Derivative of the Sine**: If $y=\sin x$, then $y'=\cos x$.

18. Thm. **Derivative of the Cosine**: If $y=\cos x$, then $y'=-\sin x$.

----------*End for Chapter 2 Quiz*----------

19. Thm. **Product Rule**: If *f(x)* and *g(x)* are differentiable functions at x, then $\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$.

20. Thm. **Extended Product Rule**: $(fgh)'=f'gh+fg'h+fgh'$.

21. Thm. **Quotient Rule**: If *f(x)* and *g(x)* are differentiable functions at x, and $g(x) \neq 0$, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$.

22. Thm. **Derivative of the Tangent**: If $y = \tan x$, then $y' = \sec^2 x$.

23. Thm. **Derivative of the Cotangent**: If $y = \cot x$, then $y' = -\csc^2 x$.

24. Thm. **Derivative of the Secant**: If $y = \sec x$, then $y' = \sec x \tan x$.

25. Thm. **Derivative of the Cosecant**: If $y = \csc x$, then $y' = -\csc x \cot x$.

*26. Thm. **Chain Rule**: If $y = f(g(x))$ is a differentiable function of $g(x)$, and $g(x)$ is a differentiable function of x, then $\frac{d}{dx}[f(g(x))] = f'(g(x)) \bullet g'(x)$.

*27. Thm. **Alternate Form of Chain Rule**: If $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$.

28. Thm. **Absolute Value Rule**: If $y = |x|$, then $y' = \frac{|x|}{x}$.

----------*End for Chapter 2 Test*----------