LIMITS AND CONTINUITY

**** Indicates that the item should be memorized in exact detail. You may be asked to quote it on a quiz or test.***

1. Def. **Even Function**: $f(x)$ is even if $f(-x) = f(x)$.

2. Def. **Odd Function**: $f(x)$ is odd if $f(-x) = -f(x)$.

3. Def. **Limit**: $\lim_{x \to c} f(x) = L$ means that $f(x)$ becomes increasingly close to L as x gets closer to c.

4. Def. **Right Hand Limit**: $\lim_{x \to c^+} f(x) = L$ means that $f(x)$ becomes increasingly close to L as x gets closer to c from the right.

5. Def. **Left Hand Limit**: $\lim_{x \to c^-} f(x) = L$ means that $f(x)$ becomes increasingly close to L as x gets closer to c from the left.

*6. Thm. **Limit Existence**: If $f(x)$ is a function and c and L are real numbers, then $\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$.

7. Thm. **Properties of Limits**: Let b and c be real numbers, let n be a positive integer, and let f and g be functions whose limit at c exists. Then

   a. Constant Function: $\lim_{x \to c} b = b$.

   b. Scalar Multiple: $\lim_{x \to c} [b \bullet f(x)] = b \bullet \lim_{x \to c} f(x)$.

   c. Sum or Difference: $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$.

   d. Product: $\lim_{x \to c} [f(x) \bullet g(x)] = \lim_{x \to c} f(x) \bullet \lim_{x \to c} g(x)$.

   e. Quotient: $\lim_{x \to c} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ provided $\lim_{x \to c} g(x) \neq 0$.

   f. Power: $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x) \right]^n$.

8. Thm. **Squeeze Law**: If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$, then $\lim_{x \to c} f(x) = L$.

9. Thm. **Trig Limit for the Sine**: $\lim_{x\to 0}\frac{\sin x}{x}=1$ for x measured in *radians*.

10. Thm. **Trig Limit for the Cosine**: $\lim_{x\to 0}\frac{1-\cos x}{x}=0$.

---*End for Chapter 1 Quiz*---

*11. Def. **Continuity at a Point**: A function $f(x)$ is continuous at c if $\lim_{x\to c} f(x)=f(c)$.

12. Def. **Continuity on an Open Interval**: A function is continuous on an open interval (a,b) if it is continuous at each point in the interval.

13. Def. **Continuity on a Closed Interval**: A function is continuous on a closed interval $[a,b]$ if it is continuous on (a,b), $\lim_{x\to a^+} f(x)=f(a)$, and $\lim_{x\to b^-} f(x)=f(b)$.

14. Def. **Discontinuity**: A function f is discontinuous at c if f is defined on an open interval containing c (except possibly at c) and f is not continuous at c.

15. Def. **Removable Discontinuity**: A discontinuity at $x=c$ is called removable if f can be made continuous by appropriately defining (or redefining) the single point $x=c$.

16. Thm. **Properties of Continuity**: If b is a real number and $f(x)$ and $g(x)$ are continuous at $x=c$, then each of the following functions are also continuous at c:

    a. Scalar Multiple: $b \bullet f(x)$

    b. Sum or Difference: $f(x)\pm g(x)$

    c. Product: $f(x)\bullet g(x)$

    d. Quotient: $\frac{f(x)}{g(x)}$ provided $g(c)\neq 0$

17. Thm. **Continuity for Polynomial and Rational Functions**: Polynomial functions are everywhere continuous. Rational functions are continuous on their domain.

18. Thm. **Continuity for Composite Functions**: If $g(x)$ is continuous at c and $f(x)$ is continuous at $g(c)$, then the composite function given by $f(g(x))$ is continuous at $x=c$.

19. Thm. **Limit of a Composite Function**: If $\lim_{x\to c} g(x)=L$ and $f(x)$ is continuous at L, then

    $$\lim_{x\to c} f(g(x))=f\left(\lim_{x\to c} g(x)\right)=f(L).$$

*20. Thm. **Intermediate Value Theorem**: If $f(x)$ is continuous on $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c)=k$.

Geometric Application: Under the given conditions, if $f(a)$ and $f(b)$ have opposite signs, then there is a point in the open interval where the graph crosses the *x*-axis.

21. Def. **Infinite Limit**: $\lim_{x\to c} f(x)=\infty$ (or $-\infty$) means that $f(x)$ increases (or decreases) with out bound as x approaches c.

22. Def. **Vertical Asymptote**: A vertical asymptote is a line $x=c$ such that $\lim_{x\to c} f(x)=\pm\infty$.

23. Def. **Limit at Infinity**: $\lim_{x\to\infty} f(x)=L$ means that $f(x)$ approaches L as x increases without bound.

24. Def. **Horizontal Asymptote**: A horizontal asymptote is a line $y=L$ such that $\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$.

25. Thm. **Limits at Infinity**: If c is any real number, then $\lim_{x\to\infty} \frac{c}{x}=0$.