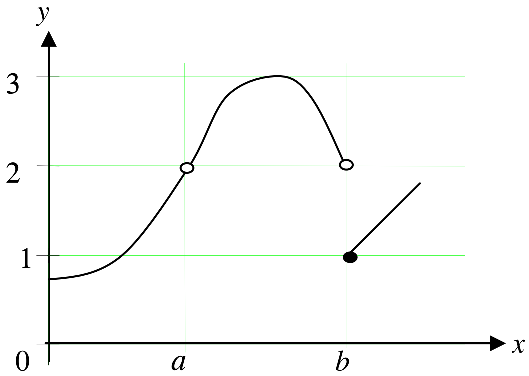
REVIEW FOR SECOND SEMESTER EXAMINATION - DAY 1

Limits, Continuity, Difference Quotients, Derivatives, Average and Instantaneous Rates of Change

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.



1. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

   A. $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$.

   B. $\lim_{x\to a} f(x) = 2$.

   C. $\lim_{x\to b} f(x) = 2$.

   D. $\lim_{x\to b} f(x) = 1$.

   E. $\lim_{x\to a} f(x)$ does not exist.

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2. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

   A. $f(0) = 0$.

   B. $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6.

   C. $-1 \le f(x) \le 3$ for all x between -3 and 6.

   D. $f(c) = 1$ for at least one c between -3 and 6.

   E. $f(c) = 0$ for at least one c between -1 and 3.

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3. The function f is defined on all the real numbers such that $f(x)=\begin{cases} x^2+kx-3 & \text{for } x\le 1 \\ 3x+b & \text{for } x>1. \end{cases}$ For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

   A. $k=-1,\ b=-3$

   B. $k=1,\ b=3$

   C. $k=1,\ b=4$

   D. $k=1,\ b=-4$

   E. $k=-1,\ b=6$

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4. $\lim\limits_{\theta\to 0}\dfrac{1-\cos\theta}{2\sin^2\theta}$ is

   A. 0 B. $\dfrac{1}{8}$ C. $\dfrac{1}{4}$ D. 1 E. nonexistent

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5. If f is a function such that $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=0$, which of the following must be true?

   A. The limit of $f(x)$ as x approaches 2 does not exist.

   B. f is not defined at $x=2$.

   C. The derivative of f at $x=2$ is 0.

   D. f is continuous at $x=0$.

   E. $f(2)=0$.

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6. If $f(x)=\sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2}<x<\dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

   A. $\dfrac{2\pi}{3}$ B. $\dfrac{3\pi}{4}$ C. $\dfrac{5\pi}{6}$ D. π E. $\dfrac{3\pi}{2}$

♦ 7. Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

A. $= 0.701$ B. -0.567 C. -0.391 D. -0.302 E. -0.258

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x	0	1	2	3
$f'(x)$	-20	-13	-7	-2

8. Let $f(x)$ be an everywhere differentiable function and $f(3) = -7$. Use the table of values above for the derivative of $f(x)$ to find an approximation for $f(3.5)$.

A. -9 B. -8 C. -6 D. -3 E. 0

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9. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

A. None B. I only C. III only D. I and III only E. II and III only

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10. Let f be the function defined by $f(x) = \begin{cases} x & x > 0 \\ x^3 & x \le 0 \end{cases}$. Which of the following statements about f is true?

A. f is an odd function.

B. f is discontinuous at $x = 0$.

C. f has a relative maximum.

D. $f'(0) = 0$.

E. $f'(x) > 0$ for $x \ne 0$.

Show all work.

11. The graph of function g is shown at the right.

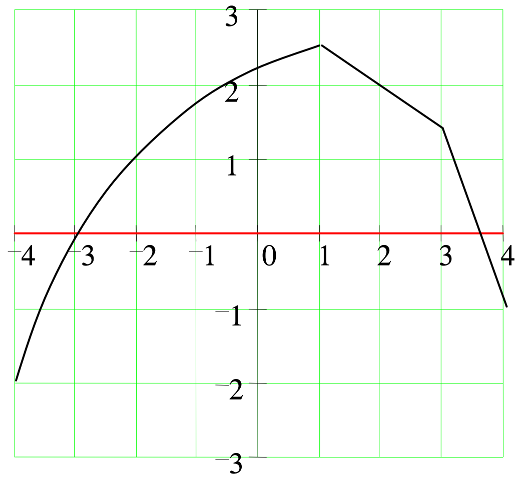
   a. Find $\lim_{x \to 1} g(x)$.

   b. Find $\lim_{h \to 0} \dfrac{g(2+h) - g(2)}{h}$.

   c. Find $\lim_{x \to -2} g'(x)$.

   d. Find $g'(g(-3))$.

   e. Find $\dfrac{dF}{dx}$ at $x = 2$, if $F(x) = g(x^2 - g(x))$.



DAY 2

Tangent Lines, Mean Value Theorem, Chain Rule, Implicit Differentiation, Differentials

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If y is a differentiable function of x, then the slope of the tangent to the curve $\tan y - 2y^2 = 5x$ at the point where $y = 0$ is

   A. 1 B. 2 C. 3 D. 4 E. 5

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2. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$

   A. -6 B. $-\frac{9}{2}$ C. $-\frac{7}{2}$ D. -3 E. $-\frac{1}{2}$

♦ 3. If $f(x) = \dfrac{e^{2x} - \ln x}{2x}$, then $f'(3.05) =$

   A. 32.214 B. 120.723 C. 122.225 D. 891.722 E. 1581.738

♦ 4. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

   A. 0 only B. 2 only C. 3 only D. 0 and 3 E. 2 and 3

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5. $\dfrac{d}{dx}\left(2^x\right) =$

   A $2^x - 1$ B. $\left(2^{x-1}\right)x$ C. $\left(2^x\right)\ln 2$ D. $\left(2^{x-1}\right)\ln 2$ E. $\dfrac{2x}{\ln 2}$

♦ 6. Administrators at Massachusetts General Hospital believe that the hospital's expenditures $E(B)$, measured in dollars, are a function of how many beds B are in use with $E(B) = 14000 + (B+1)^2$. On the other hand, the number of beds B is a function of time t, measured in days, and it is estimated that $B(T) = 20\sin\left(\frac{t}{10}\right) + 50$. At what rate are the expenditures decreasing when $t = 100$?

A. 120 dollars per day B. 125 dollars per day C. 130 dollars per day

D. 135 dollars per day E. 140 dollars per day

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7 What is the instantaneous rate of change at $x = \frac{\pi}{6}$ of the function f given by $f(x) = \tan(2x)$?

A. $\sqrt{3}$ B. $2\sqrt{3}$ C. 4 D. $4\sqrt{3}$ E. 8

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8. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

A. $g(x^2)$ B. $2xg(x)$ C. $g'(x)$ D. $2xg(x^2)$ E. $x^2 g(x^2)$

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9. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4, 3)$?

A. $-\frac{25}{27}$ B. $-\frac{7}{27}$ C. $\frac{7}{27}$ D. $\frac{3}{4}$ E. $\frac{25}{27}$

♦ 10. A tangent line to the graph of the curve $y = x\cos x$ at the point where $x = 1$ forms a right triangle with the coordinate axes. The area of the triangle is approximately

A. 1.13 B. 1.18 C. 1.21 D. 1.25 E. 1.29

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 11. Consider the curve defined by $x^2 + xy + y^2 = 27$.

a. Write an expression for the slope of the curve at any point (x, y).

b. Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

c. Find the points on the curve where the lines tangent to the curve are vertical.

DAY 3

Increasing/Decreasing, Relative and Absolute Extrema, Extreme Value Thm, Inflection Points, Concavity, Graphing

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. Let f be a function defined for all real numbers. If $f' = \frac{|4-x^2|}{x-2}$, then f is decreasing on the interval

   A. $(-\infty, 2)$ B. $(-\infty, \infty)$ C. $(-2, 4)$ D. $(-2, \infty)$ E. $(2, \infty)$

♦ 2. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

   A. One B. Three C. Four D. Five E. Seven

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3. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

   A. -3 B. $\frac{-7}{3}$ C. $\frac{-5}{2}$ D. $\frac{7}{3}$ E. $\frac{5}{2}$

♦ 4. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

   A. 0.949 B. 2.017 C. 3.106 D. 5.965 E. 8.000

♦ 5. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

   A. 0.56 B. 0.93 C. 1.18 D. 2.38 E. 2.44

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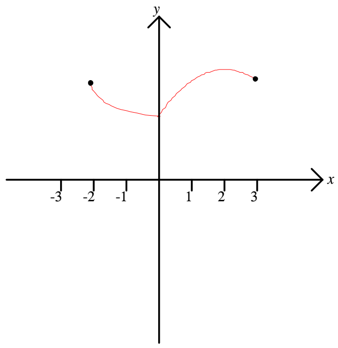
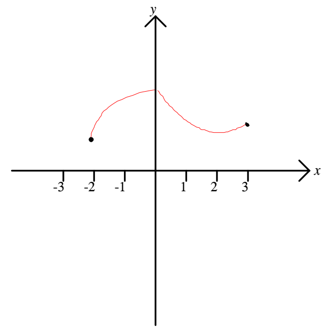
6. Let f be a function that is continuous on the closed interval [-2, 3] such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?

A.



B.



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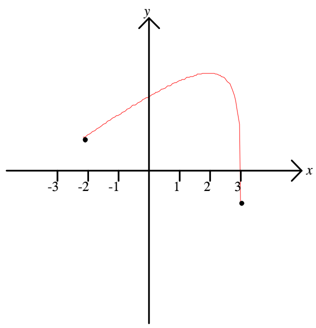

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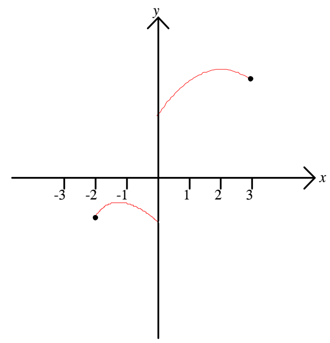
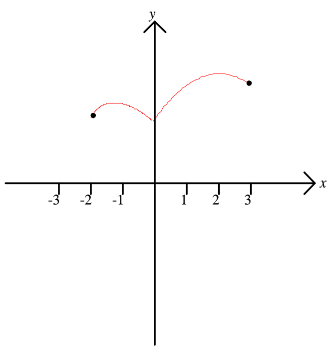


E.



♦ 7. How many inflection points does the graph of $y = \cos(3x) - \frac{1}{5}\cos(5x)$ have on the interval $0 \le x \le \pi$?

A. 1 B. 2 C. 3 D. 4 E. 5

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8. Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at $(0,1)$ and an inflection point when $x = 1$. The values of a and b are

A. $a = 1$, $b = -6$

B. $a = 1$, $b = 1$

C. $a = -6$, $b = 5$

D. $a = -6$, $b = 1$

E. $a = 6$, $b = 1$

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9. Let f be a function that has domain $[-2, 5]$. The graph of f' is shown at right. Which of the following statements are TRUE?

I. f has a relative maximum at $x = -1$.

II. f has an absolute minimum at $x = 0$.

III. f is concave down for $-2 < x < 0$.

IV. f has inflection points at $x = 0$, $x = 2$, and $x = 3$.

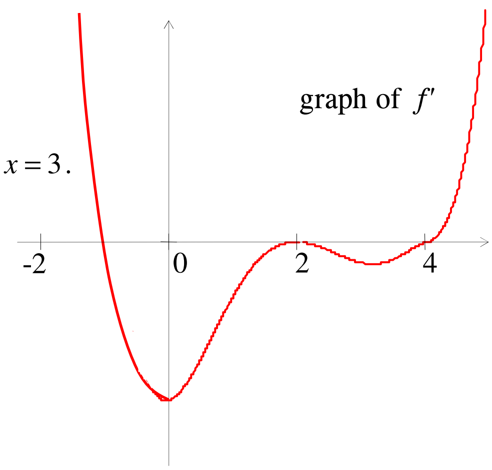
A. I, II, IV only

B. I, III, IV only

C. II, III, IV only

D. I, II, III only

E. I, II, III, IV



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10. Which of the following is true about the graph of $f(x) = \ln\left|x^2 - 4\right|$ in the interval $(-2, 2)$?

A. f is increasing.

B. f attains a relative minimum at $(0, 0)$.

C. f has a range of all real numbers.

D. f is concave down.

E. f has an asymptote at $x = 0$.

Show all work.

11. Let f be the function given by $f(x) = 2xe^{2x}$.

   a. Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

   b. Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   c. What is the range of f?

   d. Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

DAY 4

Related Rates, Distance-Velocity-Acceleration

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

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1. Let $f(x)=\sqrt{x}$. If the rate of change of f at $x=c$ is twice its rate of change at $x=1$, then $c=$

   A. $\frac{1}{4}$ B. 1 C. 4 D. $\frac{1}{\sqrt{2}}$ E. $\frac{1}{2\sqrt{2}}$

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2. A point moves on the x-axis so that its distance from the origin at time t is given by $10t-4t^2$. What is the total distance covered by the point between $t=1$ and $t=2$?

   A. 1.0 B. 1.5 C. 2.0 D. 2.5 E. 3.0

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3. A point (x, y) is moving along a curve $y=f(x)$. At the instant when the slope of the curve is $-\frac{1}{3}$, the x-coordinate of the point is increasing at the rate of 5 units per second. The rate of change, in units per second, of the y-coordinate of the point is

   A. $-\frac{5}{3}$ B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. $\frac{3}{5}$ E. $\frac{5}{3}$

♦ 4. The acceleration, $a(t)$, of a body moving in a straight line is given in terms of time t by $a(t)=4-6t$. If the velocity of the body is 20 at $t=0$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(3)-s(1)$?

   A. -10 B. 0 C. 10 D. 20 E. 30

♦ 5. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

   A. 5 in^3/sec B. 10 in^3/sec C. 15 in^3/sec D. 20 in^3/sec E. 25 in^3/sec

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♦ 6. The position of an object attached to a spring is given by $y(t)=\frac{1}{6}\cos(5t)-\frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity equal to 0?

A. 0 B. 3 C. 5 D. 6 E. 7

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7. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

A. $9t^2+1$ B. $3t^2-2t+4$ C. t^3-t^2+4t+6

D. $t^3-t^2+9t-20$ E. $36t^3-4t^2-77t+55$

♦ 8. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

A. $\frac{-7}{8}$ ft/min B. $\frac{-7}{24}$ ft/min C. $\frac{7}{24}$ ft/min D. $\frac{7}{8}$ ft/min E. $\frac{21}{25}$ ft/min

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♦ 9. Two particles start at the origin and move along the x-axis. For $0\le t\le 10$, their respective position functions are given by $x_1=\sin t$ and $x_2=e^{-2t}-1$. For how many values of t do the particles have the same velocity?

A. none B. one C. two D. three E. four

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 10. Let h be a function defined for all $x\neq 0$ such that $h(4)=-3$ and the derivative of h is given by

$h'(x)=\frac{x^2-2}{x}$ for all $x\neq 0$.

a. Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum , a local minimum, or neither at each of these values. Justify your answers.

b. On what intervals, if any, is the graph of h concave up? Justify your answer.

c. Write an equation for the line tangent to the graph of h at $x=4$.

d. Does the line tangent to the graph of h at $x=4$ lie above or below the graph of h for $x>4$? Why?

DAY 5

Riemann Sums, Trapezoidal and Midpoint Approximations, Area Approximations

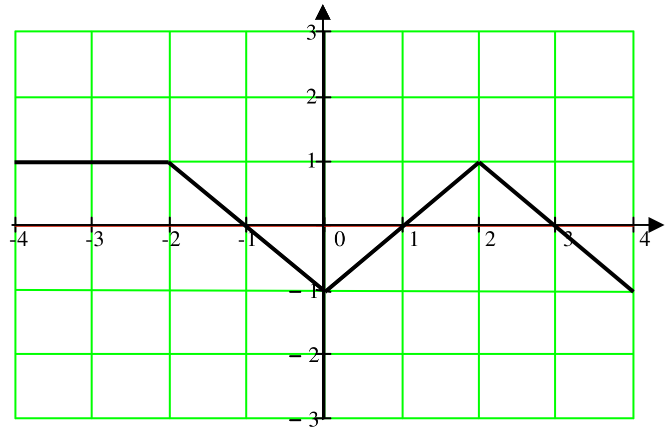
Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. The graph of f is shown at the right. Which of the following statements are true?

I. $f(2) > f'(1)$

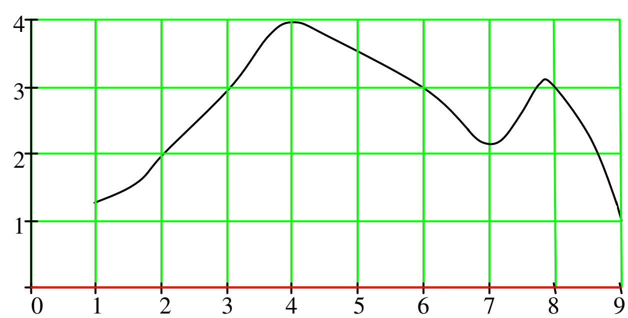
II. $\int_0^1 f(x)dx > f'(3.5)$

III. $\int_{-1}^1 f(x)dx > \int_{-1}^2 f(x)dx$



A. I only B. II only C. I and II only D. II and III only E. I, II, and III

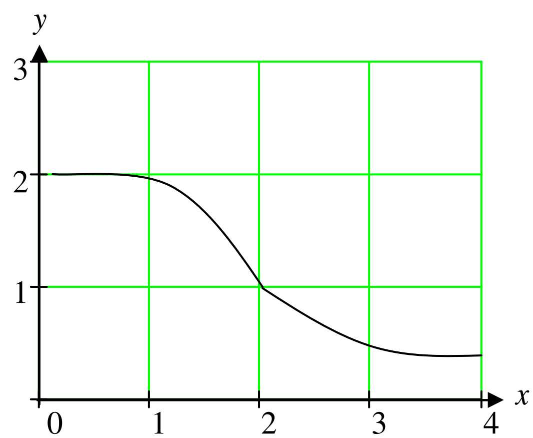
♦ 2. The graph of f over the interval $[1,9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)dx$.



A. 20 B. 21 C. 22 D. 23 E. 24

♦ 3. An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

A. 2.5 B. 3.5 C. 4.5 D. 5.5 E. 6.5



4. The graph of f is shown in the figure above. If $\int_{1}^{3} f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

A. 0.3 B. 1.3 C. 3.3 D. 4.3 E. 5.3

♦ 5. Suppose a car is moving with increasing speed according to the following table. The closest approximation of the distance traveled in the first 10 seconds is

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

A. 150 ft B. 250 ft C. 350 ft D. 450 ft E. 550 ft

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 6. Water flowed into a tank at an increasing rate $r(t)$ from $t = 0$ to $t = 5$ minutes. The rate of flow, $r(t)$, in cubic meters per minute (m^3/min), was measured at one minute intervals with the result shown in the table below.

t	0	1	2	3	4	5
$r(t)$	4	5	7	11	12	14

a. Give the best upper and lower estimates for the total amount of water that flowed into the tank for $0 \le t \le 5$. Indicate units of measure.

b. Suppose you use the average of the upper and lower estimates found in part (a) as your approximation for the total amount of water that flowed into the tank, what is the maximum error for this approximation?

c. You are now informed that for $1 \le t \le 3$ the rate of flow was exactly $r(t) = t^2 - t + 5$ m^3/min. What is the exact amount of water that flowed into the tank from $t = 1$ to $t = 3$.

♦ 7. Let $F(x) = \int_0^x \sin\left(t^2\right) dt$ for $0 \le x \le 3$.

a. Use the trapezoidal rule with four equal subdivisions of the closed interval $[0, 1]$ to approximate $F(1)$.

b. On what intervals is F increasing?

c. If the average rate of change of F on the closed interval $[1, 3]$ is k, find $\int_1^3 \sin\left(t^2\right) dt$ in terms of k.

DAY 6

Definite Integral, Integral Properties, Fundamental Theorems of Calculus, Integral as Total Change

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

   I. $\int_a^b f(x)g(x)dx = \left[\int_a^b f(x)dx\right]\left[\int_a^b g(x)dx\right]$

   II. $\int_a^b \big(f(x)+g(x)\big)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

   III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

   A. I only B. II only C. III only D. II and III only E. I, II, and III

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2. $\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)du$ is

   A. 0 B. $\dfrac{1}{2\pi}\sin x$ C. $\dfrac{1}{2\pi}\cos(2\pi x)$ D. $\cos(2\pi x)$ E. $2\pi\cos(2\pi x)$

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3. $\displaystyle\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h}$ is

   A. 0 B. 1 C. 3 D. $2\sqrt{2}$ E. nonexistent

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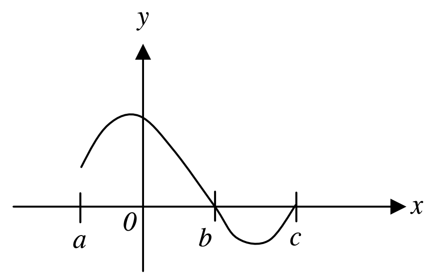
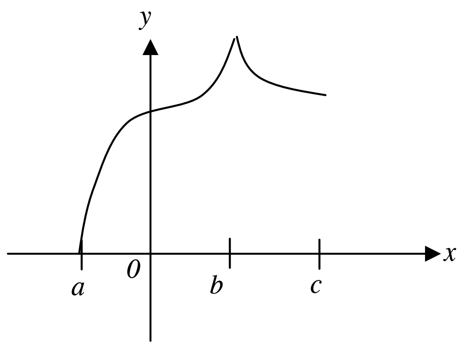
4. Let $f(x) = \int_a^x h(t)\, dt$, where h has the graph shown at right. Which of the following could be the graph of f?



A.



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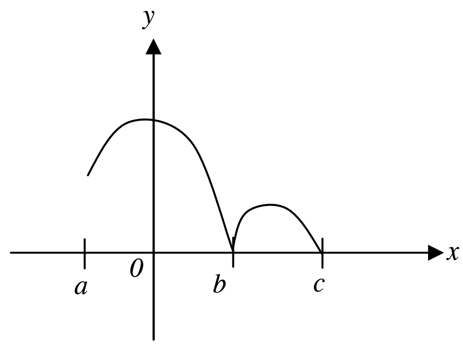

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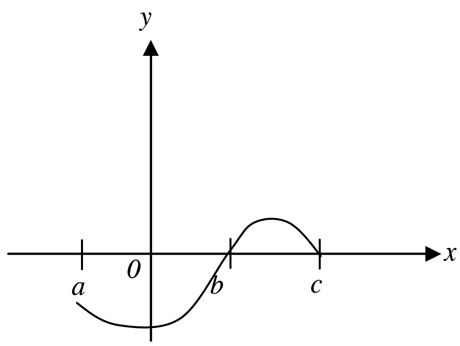
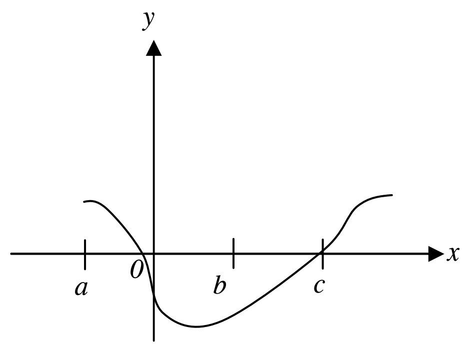


D.



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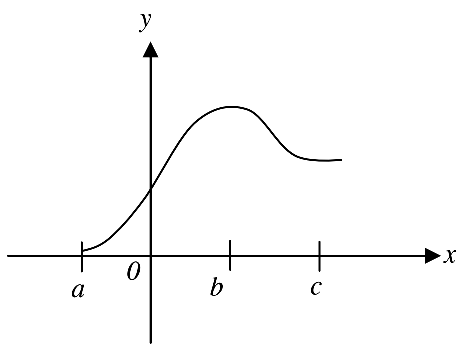


♦ 5. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

A. 54 gal B. 271 gal C. 865 gal D. 8,647 gal E. 14,778 gal

♦ 6. If $\sin(3x) - 1 = \int_a^x f(t)\,dt$, then the value of a is

A. 0 B. 1 C. -1 D. $\frac{\pi}{3}$ E. $\frac{\pi}{6}$

---

7. Which of the following are true about the function $F(x) = \int_1^x \ln(2t-1)\,dt$?

I. $F(1) = 0$

II. $F'(1) = 0$

III. $F''(1) = 1$

A. I and II only

B. I and III only

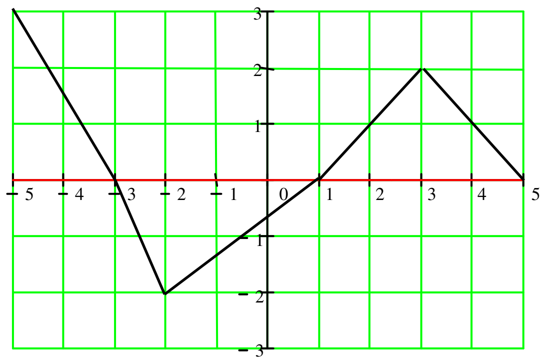
C. II and III only

D. I, II, III

E. none

Show all work.

8. Let $G(x)=\int_{-3}^{x} f(t)dt$ and $H(x)=\int_{2}^{x} f(t)dt$ where f is the function graphed below.



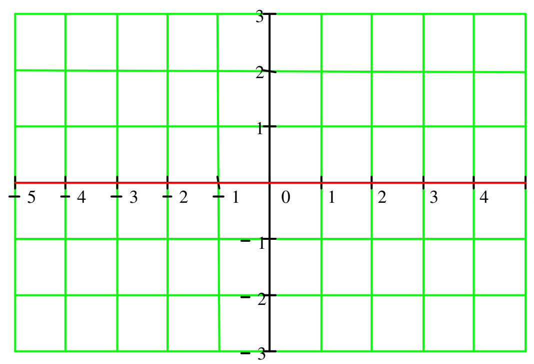
a. How are the values of $G(x)$ and $H(x)$ related? Give a geometric explanation of this relationship.

b. On which intervals of $[-5,5]$, if any, is H increasing?

c. At what x-coordinates does G have a relative maximum? Justify your answer.

d. On which subintervals of $[-5,5]$, if any, is G concave up?

e. Sketch a graph of G on the axes provided.



DAY 7

Indefinite Integral, Integration Techniques

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\int \frac{4x}{1+x^2}\,dx =$

   A. $4\,\text{Arctan}\,x + C$ B. $\frac{4}{x}\text{Arctan}\,x + C$ C. $\frac{1}{2}\ln\left(1+x^2\right) + C$

   D. $2\ln\left(1+x^2\right) + C$ E. $2x^2 + 4\ln|x| + C$

---

2. $\int_1^e \frac{x^2-1}{x}\,dx =$

   A. $e - \frac{1}{e}$ B. $e^2 - e$ C. $\frac{e^2}{2} - e + \frac{1}{2}$ D. $e^2 - 2$ E. $\frac{e^2}{2} - \frac{3}{2}$

♦ 3. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

   A. 0.048 B. 0.144 C. 5.827 D. 23.308 E. 1,640.250

---

4. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

   A. $\int_1^2 \frac{1-u^2}{u}\,du$ B. $\int_2^4 \frac{1-u^2}{u}\,du$ C. $\int_1^2 \frac{1-u^2}{2u}\,du$

   D. $\int_1^2 \frac{1-u^2}{4u}\,du$ E. $\int_2^4 \frac{1-u^2}{2u}\,du$

---

5. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

A. $2\sqrt{x^3+1}+C$ B. $\frac{3}{2}\sqrt{x^3+1}+C$ C. $\sqrt{x^3+1}+C$

D. $\ln\sqrt{x^3+1}+C$ E. $\ln\left(x^3+1\right)+C$

---

6. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx =$

A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{1}{2}\ln 2$ E. $-\ln 2$

---

7. $\int \sin(2x+3)dx =$

A. $-2\cos(2x+3)+C$ B. $-\cos(2x+3)+C$ C. $\frac{-1}{2}\cos(2x+3)+C$

D. $\frac{1}{2}\cos(2x+3)+C$ E. $\cos(2x+3)+C$

---

8. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}dx =$

A. $\arcsin\frac{x}{5}+C$ B. $\arcsin x+C$ C. $\frac{1}{5}\arcsin\frac{x}{5}+C$

D. $\sqrt{25-x^2}+C$ E. $2\sqrt{25-x^2}+C$

---

9. $\int_1^2 \frac{x^2-x}{x^3}\,dx =$

A. $\ln 2-\frac{1}{2}$ B. $\ln 2+\frac{1}{2}$ C. $\frac{1}{2}$ D. 0 E. $\frac{1}{4}$

10. $\int_{0}^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}} =$

A. $\frac{1}{2}$ B. 1 C. 2 D. $\ln 2$ E. $\text{Arctan}2 - \frac{\pi}{4}$

11. $\int_{0}^{x} \sin t\,dt =$

A. $\sin x$ B. $-\cos x$ C. $\cos x$ D. $\cos x - 1$ E. $1 - \cos x$

Show all work. Remember to show the set-up for any calculator-generated answers.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

12. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

f. Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

g. Approximate the value of your answer from part a. using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

h. Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

i. Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

DAY 8

Area Between Curves, Average Value

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

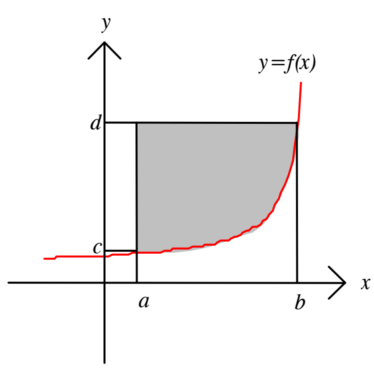
1. What is the area of the closed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$?

A. $\frac{2-e^2}{2}$ B. $\frac{e^2-3}{2}$ C. $\frac{3-e^2}{2}$ D. $\frac{e^2-2}{2}$ E. $\frac{e^2-1}{2}$

---

2. The area of the region bounded above by $y = 1 + \sec^2 x$, below by $y = 0$, on the left by $x = 0$ and on the right by $x = \frac{\pi}{4}$ is approximately

A. 1 unit^2 B. 1.25 units^2 C. 1.5 units^2 D. 1.75 units^2 E. 2 units^2



3. Which of the following represents the area of the shaded region in the figure above?

A. $\int_c^d f(y)\,dy$ B. $\int_a^b (d - f(x))dx$ C. $f'(b) - f'(a)$

D. $(b-a)[f(b) - f(a)]$ E. $(d-c)[f(b) - f(a)]$

---

4. The average value of the function $f(x) = e^{-2x}$ on the closed interval $[-1, 1]$ is

A. 0 B. $\frac{1-e^4}{e^2}$ C. $\frac{e^4-1}{e^2}$ D. $\frac{e^4-1}{4e^2}$ E. $\frac{e^4-1}{2e^2}$

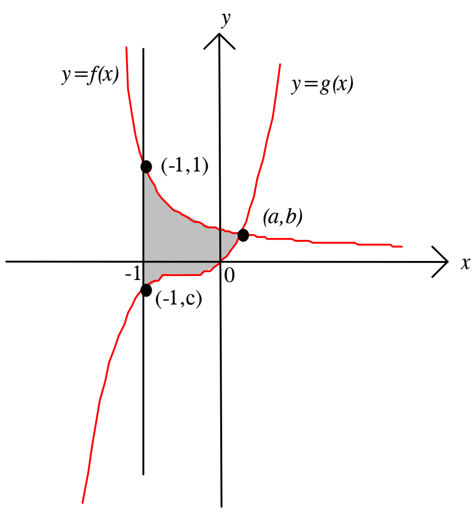
♦ 5. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

A. 1.471 B. 1.414 C. 1.277 D. 1.120 E. 0.436

---

6. The average value of $f(x) = e^{2x} + 1$ on the interval $0 \le x \le \frac{1}{2}$ is

A. e B. $\frac{e}{2}$ C. $\frac{e}{4}$ D. $2e-1$ E. $\frac{e^{2x}+1}{2}$



7. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

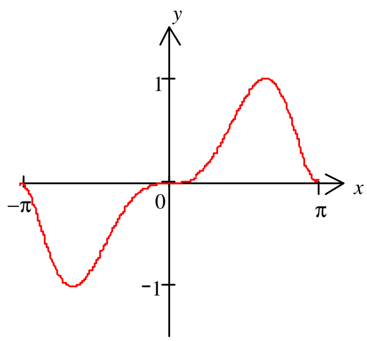
A. $\int_{-1}^{a} (|f(x)| - |g(x)|)dx$ B. $\int_{-1}^{b} g(x)dx + \int_{b}^{c} f(x)dx$ C. $\int_{-1}^{c} (f(x) - g(x))dx$

D. $\int_{-1}^{a} (f(x) - g(x))dx$ E. $\int_{0}^{a} (f(x) - g(x))dx + \int_{-1}^{0} (f(x) + g(x))dx$

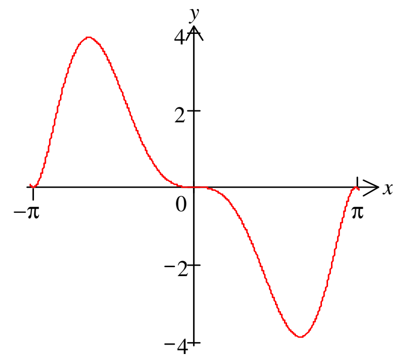
---

8. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

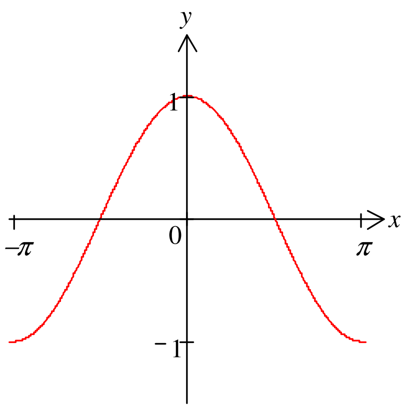
A.



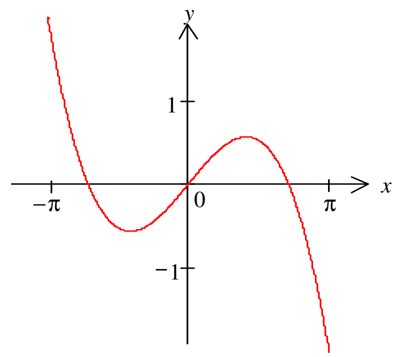
B.



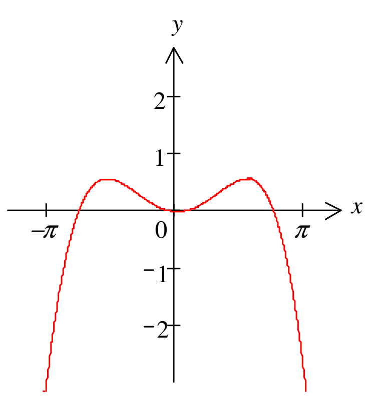
C.

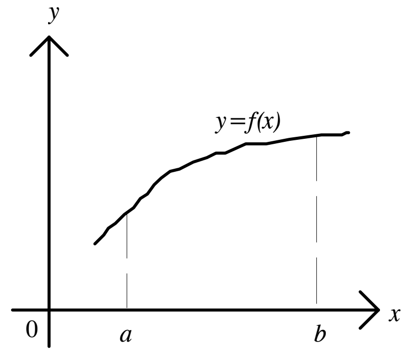


D.



E.





9. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

I. $\int_a^b f(x)dx < f(b)(b-a)$

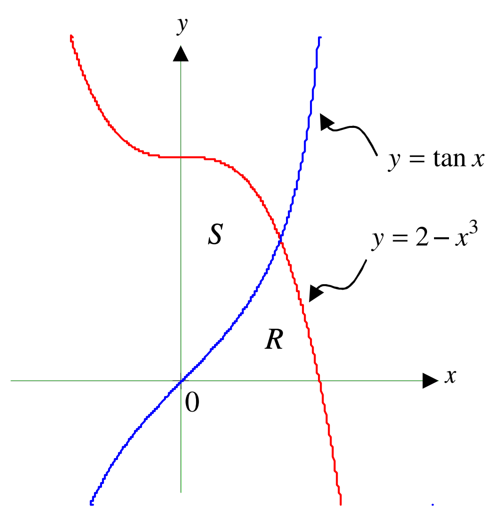
II. $\int_a^b f(x)dx < f(a)(b-a)$

III. $\int_a^b f(x)dx = f(c)(b-a)$ for some number c such that $a < c < b$.

A. I only B. II only C. III only D. I and III only E. I, II, and III

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 10. Let R and S be the regions in the first quadrant shown in the figure at right. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



a. Find area of R.

b. Find area of S.

c. Find the volume of the solid generated when S is revolved about the x-axis.

DAY 9

Optimization, Volumes of Revolution

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. Let R be the region in the first quadrant enclosed by the graph of $y=(x+1)^{\frac{1}{3}}$, the line $y=2$, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

A. $\pi\int_0^7 (x+1)^{2/3}dx$ B. $\pi\int_1^2 \left(y^3-1\right)^2 dy$ C. $\pi\int_1^2 \left(y^3-1\right)dy$

D. $\pi\int_0^7 \left(4-(x+1)^{2/3}\right)dx$ E. $\pi\int_1^2 y^2 dy$

---

2. The volume generated by revolving about the x-axis the region enclosed by the graphs of $y=2x$ and $y=2x^2$, for $0\le x\le 1$, is

A. $\pi\int_0^1 (2x-2x^2)^2 dx$ B. $\pi\int_0^1 (4x^2-4x^4)dx$ C. $2\pi\int_0^1 x(2x-2x^2)dx$

D. $\pi\int_0^2 \left(\sqrt{\frac{y}{2}}-\frac{y}{2}\right)^2 dy$ E. $\pi\int_0^2 \left(\frac{y}{2}-\frac{y^2}{2}\right)dy$

---

3. The point on the curve $2y=x^2$ nearest to $(4,1)$ is

A. $(0,0)$ B. $(2,2)$ C. $(\sqrt{2},1)$ D. $(2\sqrt{2},4)$ E. $(4,8)$

♦ 4. The region in the first quadrant enclosed by the y-axis and the graphs of $y=\cos x$ and $y=x$ is rotated about the x-axis. The volume of the solid generated is

A. 0.484 B. 0.877 C. 1.520 D. 1.831 E. 3.040

---

5. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

   A. $2\sqrt[3]{2}$ B. $2\sqrt{2}$ C. $2\sqrt[3]{4}$ D. 4 E. 8

---

6. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

   A. $\frac{\pi^2}{4}$ B. $\pi - 1$ C. π D. 2π E. $\frac{8\pi}{3}$

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 7. Let R be the first quadrant region enclosed by the graph of $y = 2e^{-x}$ and the line $x = k$.

   a. Find the area of R in terms of k.

   b. Find the volume of the solid generated when R is rotated about the x-axis in terms of k.

   c. What is the volume in part b. as $k \to \infty$?

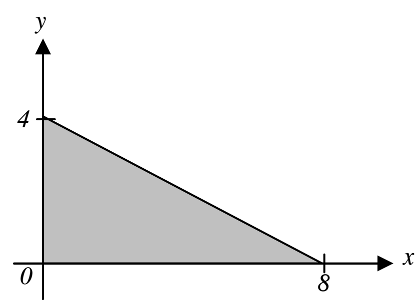
DAY 10

Volume by Slicing, Slope Fields

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

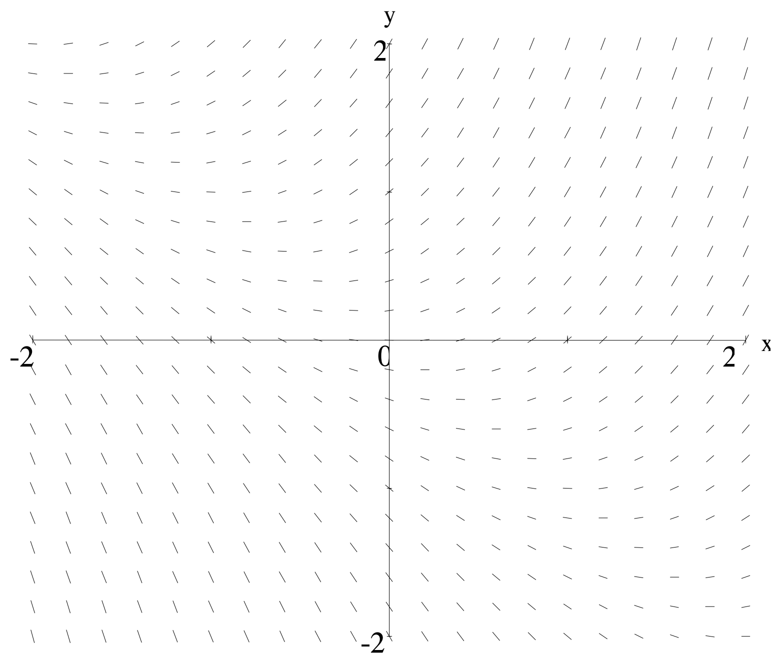
1. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the *x*-axis are squares, then its volume is

   A. $\dfrac{(1-e^{-6})}{2}$ B. $\dfrac{1}{2}e^{-6}$ C. e^{-6} D. e^{-3} E. $1-e^{-3}$



♦ 2. The base of a solid is a region in the first quadrant bounded by the *x*-axis, the *y*-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the *x*-axis are semicircles, what is the volume of the solid?

   A. 12.566 B. 14.661 C. 16.755 D. 67.021 E. 134.041



3. Shown above is a slope field for which of the following differential equations?

A. $\dfrac{dy}{dx} = 1 + x$ B. $\dfrac{dy}{dx} = x^2$ C. $\dfrac{dy}{dx} = x + y$ D. $\dfrac{dy}{dx} = \dfrac{x}{y}$ E. $\dfrac{dy}{dx} = \ln y$

---

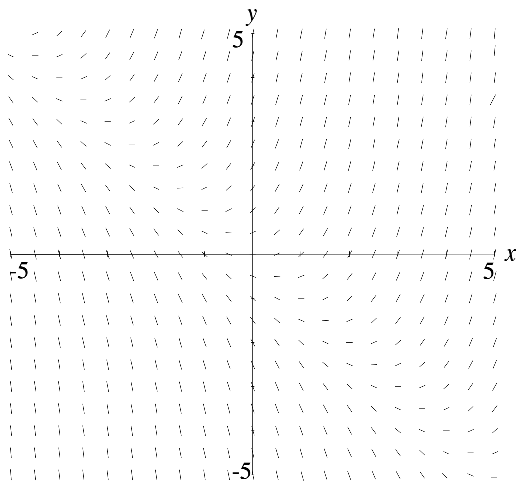
4. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

A. $\pi\int_0^2 (2 - y)^2\,dy$ B. $\int_0^2 (2 - y)\,dy$ C. $\pi\int_0^{\sqrt{2}} \left(2 - x^2\right)^2 dx$

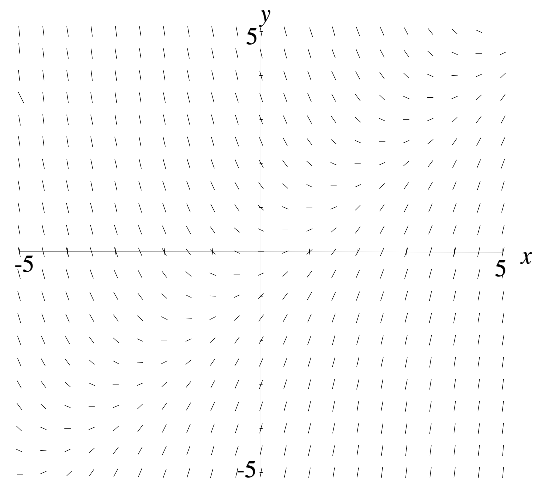
D. $\int_0^{\sqrt{2}} \left(2 - x^2\right)^2 dx$ E. $\int_0^{\sqrt{2}} \left(2 - x^2\right) dx$

5. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

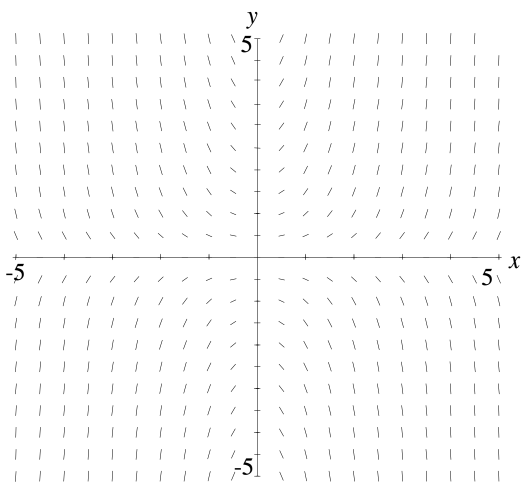
A.



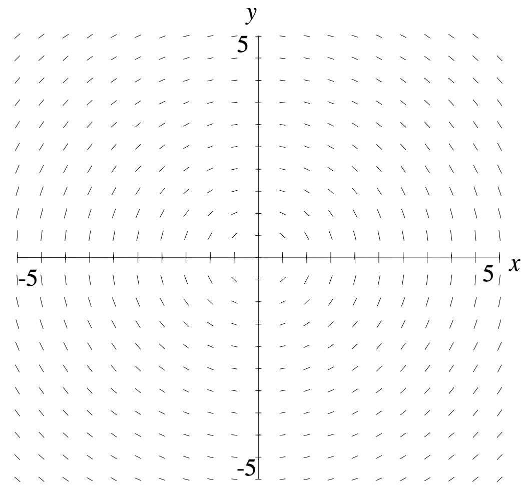
B.



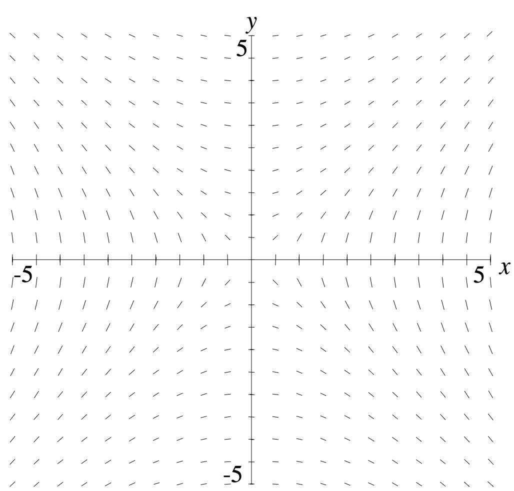
C.



D.



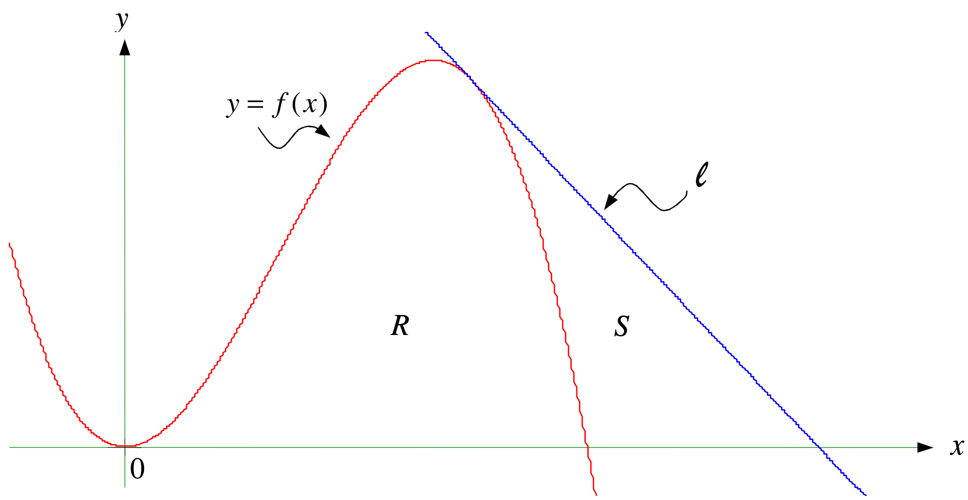
E.



♦ 6. Let the base of a solid be the first quadrant region enclosed by the x-axis and one arch of the graph of $y = \sin x$. If all cross sections perpendicular to the x-axis are squares, then the volume of the solid is approximately

A. 0.52 units^3 B. 0.79 units^3 C. 1.05 units^3 D. 1.57 units^3 E. 2.00 units^3

Show all work. Remember to show the set-up for any calculator-generated answers.



♦ 7. Let f be the function given by $f(x) = 4x^2 - x^3$, and let l be the line $18 - 3x$, where l is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line l, and the x-axis, as shown above.

a. Show that l is tangent to the graph of $y = f(x)$ at the point $x = 3$.

b. Find the area of S.

c. Find the volume of the solid generated when R is revolved about the x-axis.

DAY 11

Differential Equations and Inverse Functions

Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

   A. $e^{\tan x} + 4$ B. $e^{\tan x} + 5$ C. $5e^{\tan x}$ D. t[illegible] E. $\tan x + 5e^x$

---

2. Let $f(x) = x^3 + x$. If h is the inverse function of f, then $h'(2) =$

   A. $\frac{1}{13}$ B. $\frac{1}{4}$ C. 1 D. 4 E. 13

---

3. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

   A. $\frac{-2}{3}$ B. $\frac{-1}{3}$ C. 0 D. $\frac{1}{3}$ E. $\frac{2}{3}$

♦ 4. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

   A. 4.2 pounds B. 4.6 pounds C. 4.8 pounds D. 5.6 pounds E. 6.5 pounds

---

5. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

   A. $2e^{kty}$ B. $2e^{kt}$ C. $e^{kt} + 3$ D. $kty + 5$ E. $\frac{1}{2}ky^2 + \frac{1}{2}$

---

6. A solution of the equation $\frac{dy}{dx} + 2xy = 0$ that contains the point $(0, e)$ is

A. $y = e^{1-x^2}$ B. $y = e^{1+x^2}$ C. $y = e^{1-x}$ D. $y = e^{1+x}$ E. $y = e^{x^2}$

♦ 7. If $f(x) = 2x + \sin x$ and the function g is the inverse of f, then $g'(2) =$

A. 0.32 B. 0.34 C. 0.36 D. 0.38 E. 0.30

♦ 8. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

A. 0.069 B. 0.200 C. 0.301 D. 3.322 E. 5.000

Show all work. Remember to show the set-up for any calculator-generated answers.

♦ 9. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

a. Write an equation for y, the amount of oil remaining in the well at any time t.

b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

c. In order not to lose money, at what time t should oil no longer be pumped from the well?

♦ 10 Let f be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $\frac{-\pi}{4} < x < \frac{\pi}{2}$.

a. Write an equation for the line tangent to the graph of f at the point where $x = 0$.

b. Using the equation found in part a., approximate $f(0.02)$.

c. Let f^{-1} denote the inverse function of f. Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1}.