

REVIEW FOR FIRST SEMESTER EXAMINATION – DAY 1

Limits, Continuity, Difference Quotients, Derivatives, Average and Instantaneous Rates of Change

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. If f is continuous for $a \leq x \leq b$, and differentiable for $a < x < b$, which of the following could be false?

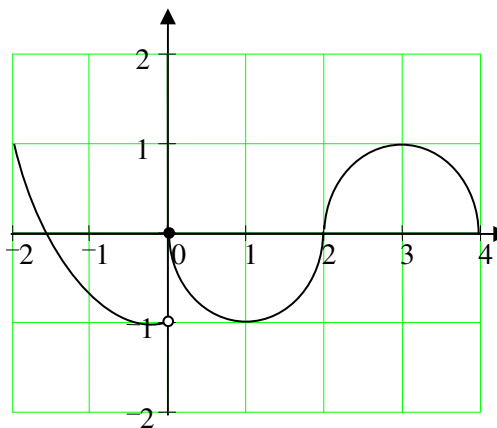
- A. $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
- B. $f'(c) = 0$ for some c such that $a < c < b$.
- C. f has a minimum value on $a \leq x \leq b$.
- D. f has a maximum value on $a \leq x \leq b$.
- E. $\int_a^b f(x) dx$ exists.

2. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln x & \text{for } 2 < x \leq 4 \end{cases}$, then $\lim_{x \rightarrow 2} f(x)$ is

- A. $\ln 2$
- B. $\ln 8$
- C. $\ln 16$
- D. 4
- E. nonexistent

3. The graph of the function f shown in the figure at right has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x , $-2 < x < 4$, is f not differentiable?

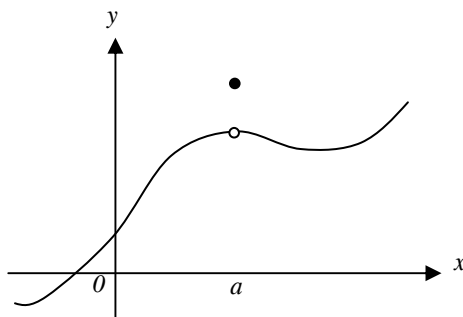
- A. 0 only
- B. 0 and 2 only
- C. 1 and 3 only
- D. 0, 1, and 3 only
- E. 0, 1, 2, and 3



- ◆ 4. Let f be the function given by $f(x) = 2(3-x)^{3/2}$ and let g be the function given by $g(x) = -3x^2$. At what value(s) of x do the graphs of f and g have parallel tangent lines?
- A. $\frac{3}{4}$ B. 0 C. $-\frac{1}{2}$ and $\frac{1}{2}$ D. -1 and $\frac{3}{4}$ E. none

x	0	1	2
$f(x)$	1	k	2

5. The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$
- A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. 3



- ◆ 6. The graph of a function f is shown above. Which of the following statements about f is false?
- A. f is continuous at $x = a$.
- B. f has a relative maximum at $x = a$.
- C. $x = a$ is in the domain of f .
- D. $\lim_{x \rightarrow a^+} f(x)$ is equal to $\lim_{x \rightarrow a^-} f(x)$.
- E. $\lim_{x \rightarrow a} f(x)$ exists.

- ◆ 7. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?
- I. f is continuous at $x = 0$.
 - II. f is differentiable at $x = 0$.
 - III. f has an absolute minimum at $x = 0$.
- A. I only B. II only C. III only D. I and III only E. II and III only
-

- ◆ 8. If $a \neq 0$, then $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$ is
- A. $\frac{1}{a^2}$ B. $\frac{1}{2a^2}$ C. $\frac{1}{6a^2}$ D. 0 E. nonexistent
-

- ◆ 9. Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?
- I. f has at least 2 zeros.
 - II. The graph of f has at least one horizontal tangent.
 - III. For some c , $2 < c < 5$, $f(c) = 3$.
- A. None B. I only C. I and II only D. I and III only E. I, II, and III
-
-

Show all work. Remember to show the set-up for any calculator-generated answers.

- ◆ 10. Let p and q be real numbers and let f be the function defined by
- $$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2 & \text{for } x \leq 1 \\ qx + p & \text{for } x > 1 \end{cases}$$
- a. Find the value of q , in terms of p , for which f is continuous at $x = 1$.
 - b. Find the values of p and q for which f is differentiable at $x = 1$.
 - c. If p and q have the values determined in part b, is f'' a continuous function? Justify your answer.
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DAY 2

Tangent Lines, Differentials, Differentiation Rules, Chain Rule, Implicit Differentiation

Multiple Choice. Questions marked with an \blacklozenge allow use of a calculator. Choose the one correct answer.

1. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

- A. $\frac{-7}{2}$ B. -2 C. $\frac{2}{7}$ D. $\frac{3}{2}$ E. $\frac{7}{2}$
-

2. Let f and g be differentiable functions with the following properties:

- i. $g(x) > 0$ for all x .
ii. $f(0) = 1$.

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

- A. $f'(x)$ B. $g(x)$ C. $x+1$ D. 0 E. 1
-

3. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

- A. -2 B. $\frac{1}{6}$ C. $\frac{1}{2}$ D. 2 E. 6
-

4. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

- A. $\sqrt{3}$ B. $2\sqrt{3}$ C. 4 D. $4\sqrt{3}$ E. 8
-

5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

- A. $y = 2x + 1$ B. $y = x + 1$ C. $y = x$ D. $y = x - 1$ E. $y = 0$

-
6. If $\frac{dy}{dt} = kt$ and k is a nonzero constant, then y could be
- A. kt^2 B. $\frac{k}{2}t^2 - 8$ C. $2kt^2$ D. $\frac{1}{2}k^2t + 1$ E. $kt - 5$
-

x	0	2	4	6
$f'(x)$	-2	2	3	4

7. Let $f(x)$ be an everywhere differentiable function and $f(6) = -5$. Use the table of values above for the derivative of $f(x)$ to find an approximation for $f(6.5)$.

- A. -20 B. -4.75 C. -4.5
D. -3 E. 1
-

- ◆ 8. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?
- A. $y = 8x - 5$ B. $y = x + 7$ C. $y = x + 0.763$
D. $y = x - 0.122$ E. $y = x - 2.146$
-

Show all work. Remember to show the set-up for any calculator-generated answers.

- ◆ 9. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.
- a. Find the slope of the graph of f at the point where $x = 1$.
- b. Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
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DAY 3

Relative and Absolute Extrema, Inflection Points, Concavity

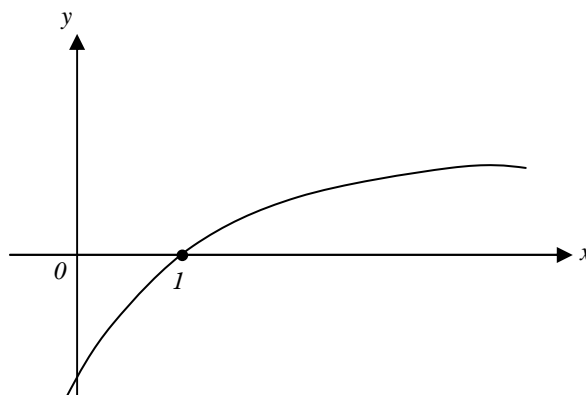
Multiple Choice. Questions marked with an \blacklozenge allow use of a calculator. Choose the one correct answer.

1. What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

- A. 5 B. 0 C. $-\frac{10}{3}$ D. -5 E. -10
-

2. The graph of a twice-differentiable function f is shown in the figure below. Which of the following is true?

- A. $f(1) < f'(1) < f''(1)$
B. $f(1) < f''(1) < f'(1)$
C. $f'(1) < f(1) < f''(1)$
D. $f''(1) < f(1) < f'(1)$
E. $f''(1) < f'(1) < f(1)$



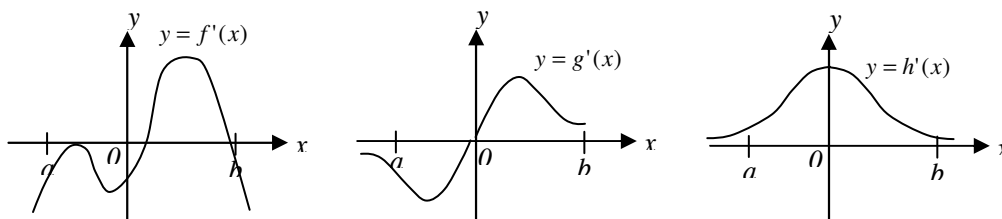
3. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

- A. -1 only B. 2 only C. -1 and 0 only
D. -1 and 2 only E. -1, 0, and 2 only
-

4. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

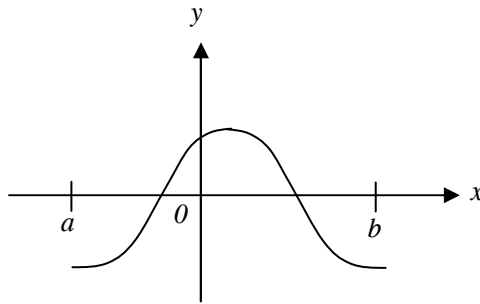
- A. $\left(\frac{-1}{2}, \infty\right)$ B. $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ C. $(0, \infty)$ D. $(-\infty, 0)$ E. $\left(-\infty, \frac{-1}{\sqrt{2}}\right)$
-

- ◆ 5. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?
- A. One B. Three C. Four D. Five E. Seven
-



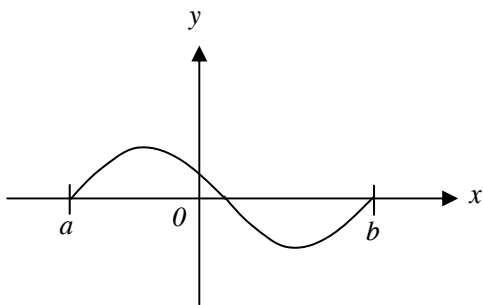
- ◆ 6. The graphs of the *derivatives* of the functions f , g , and h are shown above. Which of the functions f , g , or h have a relative maximum on the open interval $a < x < b$?
- A. f only B. g only C. h only D. f and g only E. f , g , and h
-

- ◆ 7. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?
- A. f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
- B. f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
- C. f has relative minima at $x = -2$ and at $x = 2$.
- D. f has relative maxima at $x = -2$ and at $x = 2$.
- E. It cannot be determined if f has any relative extrema.
-

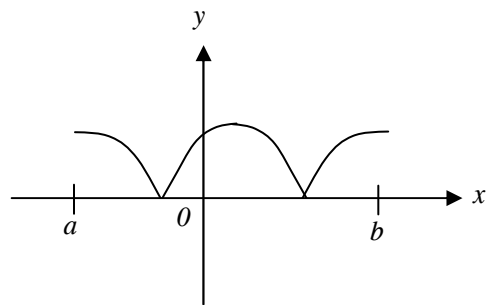


8. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f ?

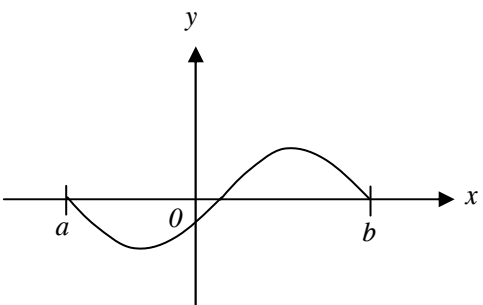
A.



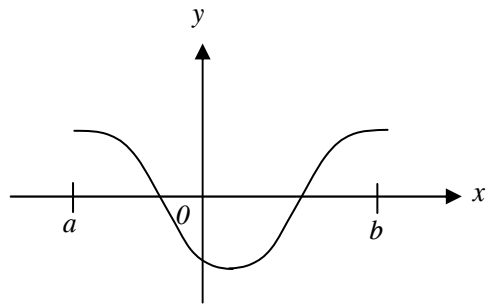
B.



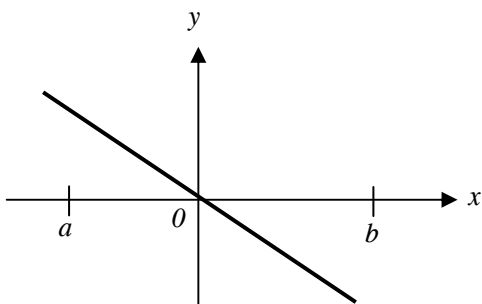
C.



D.



E.



Show all work. Remember to show the set-up for any calculator-generated answers.

- ◆ 9. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
- Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.
 - Write an equation of each horizontal tangent line to the curve.
 - The line through the origin with slope -1 is tangent to the curve at point P . Find the x - and y -coordinates of point P .
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DAY 4

Related Rates, Max/Min Word Problems, Velocity/Acceleration, Differential Equations

Multiple Choice. Questions marked with an \blacklozenge allow use of a calculator. Choose the one correct answer.

1. A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

A. 1 B. 2 C. 3 D. 4 E. 5

2. The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

A. 9 B. 12 C. 14 D. 21 E. 40

- \blacklozenge 3. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C , what is the rate of change of the area of the circle, in square centimeters per second?

A. $-(0.2)\pi C$ B. $-(0.1)C$ C. $\frac{-(0.1)C}{2\pi}$ D. $(0.1)^2 C$ E. $(0.1)^2 \pi C$

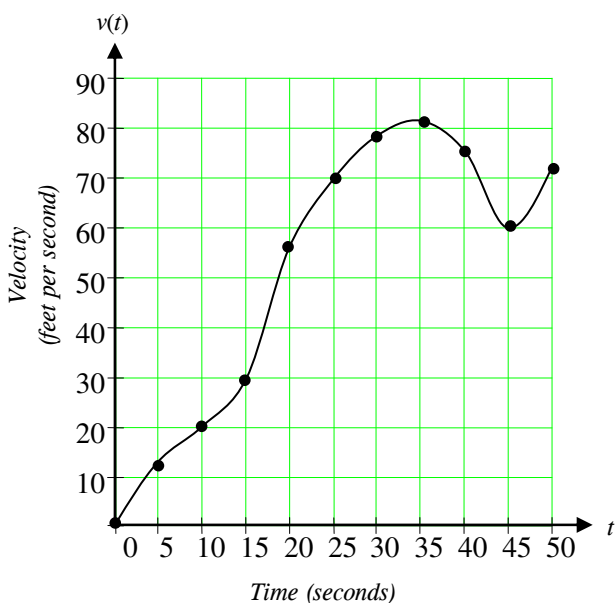
- \blacklozenge 4. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

A. A is always increasing.
B. A is always decreasing.
C. A is decreasing only when $b < h$.
D. A is decreasing only when $b > h$.
E. A remains constant.

- ◆ 5. Let $F(x)$ be an antiderivative of $\frac{2-x}{x^3}$. If $F(1) = 2$, then $F(9) =$

A. 1.986 B. 2.099 C. 10 D. 17.111 E. -240.333

Show all work. Remember to show the set-up for any calculator-generated answers.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

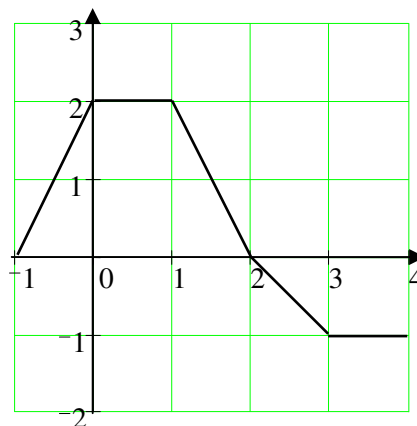
- ◆ 6. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
 - Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.
 - Find one approximation for the acceleration of the car, in ft/sec^2 , at $t = 40$. Show the computations you used to arrive at your answer.
 - Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

DAY 5

Riemann Sums, Trapezoidal/Midpoint Rules, Area, Properties of Integration

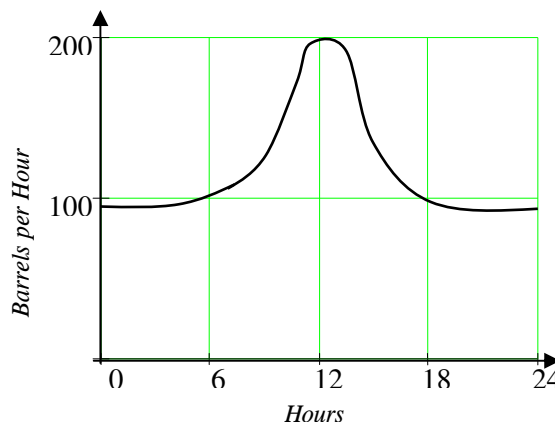
Multiple Choice. Questions marked with an \blacklozenge allow use of a calculator. Choose the one correct answer.

1. The graph of a piecewise-linear function f , for $-1 \leq x \leq 4$, is shown at right. What is the value of $\int_{-1}^4 f(x) dx$?



- A. 1 B. 2.5 C. 4
 D. 5.5 E. 8

2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown at right. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?



- A. 500 B. 600 C. 2,400
 D. 3,000 E. 4,800

3. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$

- A. 0 B. 1 C. $\frac{ab}{2}$ D. $b - a$ E. $\frac{b^2 - a^2}{2}$

4. What are all values for k for which $\int_{-3}^k x^2 dx = 0$?

- A. -3 B. 0 C. 3 D. -3 and 3 E. $-3, 0,$ and 3
-

x	2	5	7	8
$f(x)$	10	30	40	20

5. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal

approximation of $\int_2^8 f(x)dx$?

- A. 110 B. 130 C. 160 D. 190 E. 210
-

Show all work. Remember to show the set-up for any calculator-generated answers.

- ◆ 6. Let R be the region bounded by the x -axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
- Find the area of the region R .
 - Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
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DAY 6

Techniques of Integration, Fundamental Theorems, Applications of Integration

Multiple Choice. Questions marked with an ♦ allow use of a calculator. Choose the one correct answer.

1. $\int_1^2 \frac{1}{x^2} dx =$

- A. $-\frac{1}{2}$ B. $\frac{7}{24}$ C. $\frac{1}{2}$ D. 1 E. $2 \ln 2$
-

2. $\int_0^x \sin t dt =$

- A. $\sin x$ B. $-\cos x$ C. $\cos x$
D. $\cos x - 1$ E. $1 - \cos x$
-

3. $\int \frac{x^3 - 1}{x^2} dx =$

- A. $\frac{1}{2}x^2 - \frac{1}{3x^3} + C$ B. $\frac{\frac{1}{4}x^4 - x}{\frac{1}{3}x^3} + C$ C. $1 + \frac{1}{x^3} + C$
D. $\frac{1}{2}x^2 + \frac{1}{3x^3} + C$ E. $\frac{1}{2}x^2 + \frac{1}{x} + C$
-

4. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

- A. -3 B. -2 C. 2 D. 3 E. 18
-

5. If $\frac{dy}{dt} = \frac{t^2 - 6t}{\sqrt{t}}$, then y could be

- A. $\frac{\frac{1}{3}t^3 - 3t^2}{\frac{2}{3}t^{3/2}}$ B. $\frac{t^3 - 6t^2}{t^{1/2}}$ C. $t^{5/2} - 6t^{3/2}$
D. $\frac{2}{5}t^{5/2} - 4t^{3/2} - 5$ E. $\frac{3}{2}t^{1/2} - 3t^{-1/2}$
-

6. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x , then $\int_1^3 f(2x)dx =$

- A. $2F(3) - 2F(1)$ B. $\frac{1}{2}F(3) - \frac{1}{2}F(1)$ C. $2F(6) - 2F(2)$
D. $F(6) - F(2)$ E. $\frac{1}{2}F(6) - \frac{1}{2}F(2)$
-

7. What is the average value of $y = x^2\sqrt{x^3 + 1}$ on the interval $[0, 2]$?

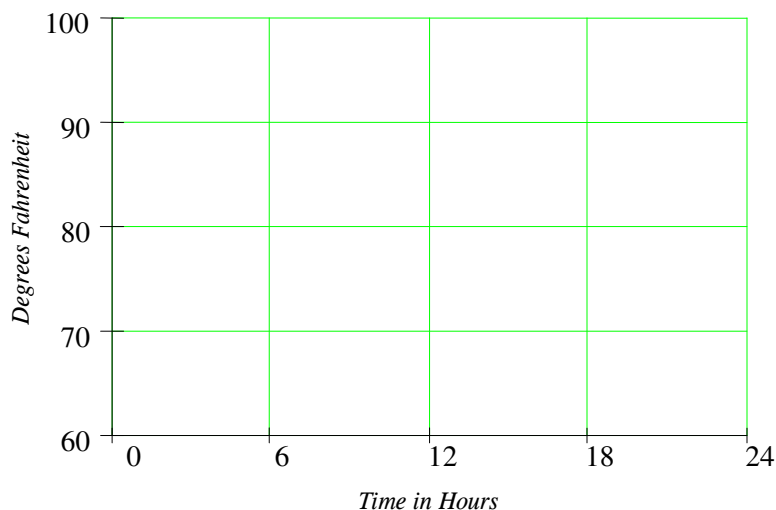
- A. $\frac{26}{9}$ B. $\frac{52}{9}$ C. $\frac{26}{3}$ D. $\frac{52}{3}$ E. 24
-

◆ 8. If $0 \leq k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

- A. 1.471 B. 1.414 C. 1.277 D. 1.120 E. 0.436
-
-

Show all work. Remember to show the set-up for any calculator-generated answers.

- ◆ 9. The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right)$, $0 \leq t \leq 24$, where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.
- a. Sketch the graph of F on the grid below.



- b. Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.
- c. An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
- d. The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?
-