VOLUMES OF SOLIDS OF REVOLUTION – SHELL METHOD I

1. Given a region bounded by the curve $y = x^2$, $y = 0$, and $x = 3$. Find the volume of the solid formed by revolving the region about the y-axis.

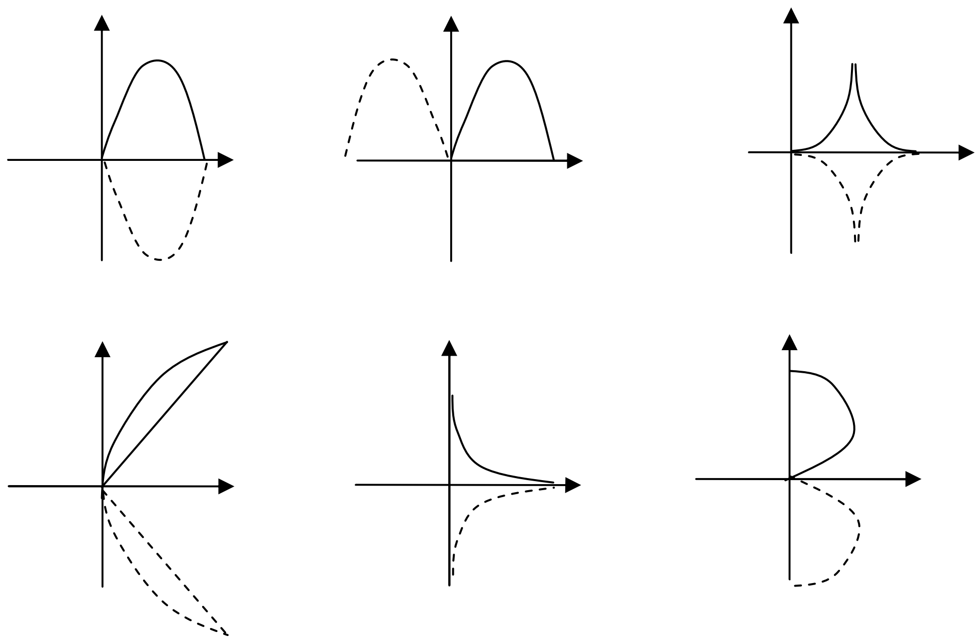
2. Given a region bounded by the curve $y^2 = 12 - 4x$, $y = 0$, $x = 0$, and $y = 2$. Find the volume of the solid formed by revolving the region about the x-axis.

3. Let R be the region enclosed by the graphs of $y = e^x$, and $y = (x-1)^2$. Find the volume of the solid generated when R is revolved about the y-axis.

4. Given the region bounded by the curves $y = \ln x$, $y = \frac{x}{4}$, and the x-axis. Find the volume of the solid formed when the region is revolved about the x-axis.

VOLUMES OF SOLIDS OF REVOLUTION – SHELL METHOD II

5. Choose the Best Method for Finding the Volume of the Solid of Revolution



6. Given a region in the *second* quadrant bounded by the curve $y = x^2$, $y = \frac{-x}{2} + 3$, and $x = 0$.
Find the volume of the solid formed by revolving the region about the line $x = 1$.

7. Given a region bounded by the curve $x - y = -1$, $y = 0$, and $x = 0$. Find the volume of the solid formed by revolving the region about the line $y = -1$.

8. Given a region bounded by the curve $y = \dfrac{1}{\sqrt{x}}$, $y = 0$, and the lines $x = 4$ and $x = 9$. Find the volume of the solid formed by revolving the region about the line $x = 10$.

9. Given a region bounded by the curve $y = \dfrac{4}{x+1}$, $x = 3$, and the x-axis, and the y-axis. Find the volume of the solid formed by revolving the region about the line $x = -1$.

ARC LENGTH

10. What is meant by the length of an arc?

11. How would we approximate the length of an arc?

12. Suppose we subdivide an arc into n pieces. Let Δs_i represent one subdivision of the arc length.

    Then $\Delta s_i \approx \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$ by ____

    a. Adding them all up gives an approximation for the arc length s:

    $$s \approx \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

    b. Now rewrite the radical expression by factoring out $(\Delta x_i)^2$:

    $$s \approx \sum_{i=1}^{n} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

    c. By the Mean Value Theorem there exists a number $c \in (x_{i-1}, x_i)$ such that $f'(c_i) = \dfrac{y_i - y_{i-1}}{x_i - x_{i-1}}$ or

    $f'(c_i) = \dfrac{\Delta y_i}{\Delta x_i}$.

    d. Substituting this into the summation we get:

    $$s \approx \sum_{i=1}^{n} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

13. To get an exact value, we take the limit of the summation as the norm of the partition approaches zero.

    Thus $s = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{1 + [f'(c_i)]^2}\,(\Delta x_i)$

    or $s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

14. Find the length of the arc of $y = 2(x-1)^{\frac{3}{2}} - 4$ from $x = 2$ to $x = 5$.

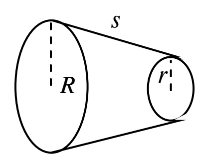
15. Find the length of the arc of $x = \frac{1}{3}(y-3)\sqrt{y}$ from $0 \le y \le 3$.

16. Find the length of the arc of $y = x^{\frac{2}{3}}$ from (1, 1) to (8, 4).

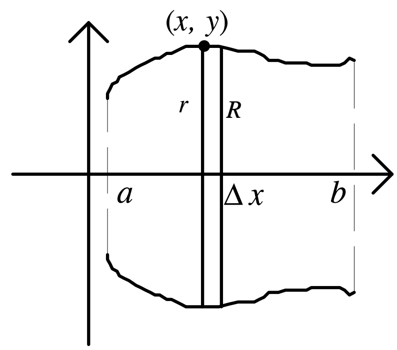
SURFACE AREA

17. How would we approximate the area of a surface of revolution?

18. At right is illustrated a frustum of a cone. The surface area of a frustum of a cone is found by the formula $\pi(R + r)s$



19. Suppose that we slice a surface of revolution into circular slices, just as we would slice a solid of revolution.



   a. The surface area of each subdivision approximates that of the lateral area of a frustum of a cone. Hence we use the formula:

   $\Delta SA \approx \pi(R + r)\Delta s$

   b. Since the two radii are almost the same (for thin slices) we will use the same value for each.

   $\Delta SA \approx \pi(2 \bullet r)\Delta s$

   c. Since Δs corresponds to the length of the arc in the subdivision, by substitution (using the arc length formula in the previous lesson) we have:

   $\Delta SA \approx$ ______________________________

20. To get an exact value for the surface area, we take the limit of the summation as the norm of the partition approaches zero.

   $SA =$ ________________________________

   To emphasize that the radius must be expressed as a function of x, we normally write the formula as:

   $$SA = 2\pi\int_a^b r(x)\sqrt{1 + [f'(x)]^2}\,dx$$

21. Find the area of the surface obtained by revolving $x^2 + y^2 = 4$ about the *x*-axis from $\frac{1}{2} \le x \le 1$.

22. Find the area of the surface obtained by revolving $y = x^2$ about the *y*-axis from $0 \le x \le 1$

23. Find the area of the surface obtained by revolving $x = \frac{1}{4}y^2$ about the *x*-axis from $1 \le x \le 4$.
Use TI-89 to evaluate.

WORK DONE IN LIFTING A CHAIN

24. A 20 foot chain weighing 1/2 lb/ft hangs from the top of a tall building. How much work is done in pulling the chain to the roof of the building?

25. A 50 foot chain weighing 2 lb/ft is hanging vertically. How much work is done in lifting one end of the chain to the level of the other end?

26. A steam shovel is excavating sand. Each load weighs 500 lb when excavated. The shovel lifts each load to a height of 15 feet in $\frac{1}{2}$ min, then dumps it. A leak in the shovel lets sand drop out while the shovel is being raised. The rate at which the sand leaks out is 160 lb/min. Find the amount of work done in raising one load.

27. A 20 foot chain weighing 1/2 lb/ft hangs from the top of a 10 foot building (with 10 feet of chain on the ground) How much work is done in pulling the chain to the roof of the building?

FLUID FORCE

28. The face of a dam adjacent to the water has the shape of an isosceles trapezoid of altitude 20 feet, upper base 50 feet and lower base 40 feet. Find the total force exerted by the water on the dam when the water is 15 feet deep.

29. The vertical ends of a trough are in the shapes of semicircles of diameter 2 feet with diameter horizontal. Find the total force on one end when the trough is full of water.

30. Find the fluid force on a vertical side of a tank if the tank is full of water and the side has the shape of a right triangle whose legs are 5 feet and 10 feet. The longer leg is positioned horizontally at the bottom of the tank.

FLUID FORCE CONTINUED

31. A vertical dam contains a rectangular gate 3 feet wide and 2 feet high. The top of the gate is horizontal and 10 feet below the water surface. Find the total force on the gate.

32. A dam has a vertical gate in the shape of an isosceles trapezoid with upper base 6 feet, lower base 8 feet, and height 3 feet. If the gate can withstand a force of 15,160 lb, what is the highest level above the top of the gate to which the dam can be filled?

33. A dam has the shape of a parabola 40 feet high and 40 feet across the top. Find the force on the dam when it is filled to a level 10 feet below the top.

INTEGRATION BY PARTS – I

34. $\int x^7 \sqrt{5 - x^4}\, dx =$

35. $\int x \operatorname{arcsec} x\, dx =$ for $x > 0$

36. $\int 9x \cot^2(3x)\, dx =$

37. $\int \text{arccot}\, x\,dx =$

38. $\int_1^e x^2 \ln x\,dx =$

INTEGRATION BY PARTS – II

39. $\int x^2 e^x \, dx =$

40. $\int e^{-x} \cos x dx =$

41. $\int \sec^3 x dx =$

42. $\int_0^{\frac{\pi}{4}} \cos^2 x dx =$

PARTIAL FRACTIONS

43. $\int \frac{1}{1-x^2}dx =$

44. $\int \frac{x^3 - x^2 + 2x + 3}{x^2 + 3x + 2}dx =$

45. $\int \frac{1}{x^3 - x^2}dx =$

46. $\int \frac{\cos x dx}{\sin^2 x - 2\sin x - 3} =$

47. $\int \frac{e^t dt}{e^{2t} + 3e^t + 2} =$

48. $\int_{-1}^{0} \frac{x^2 + 1}{(x-1)^3} dx =$

REVIEW: APPLICATIONS OF INTEGRATION

49. Find the length of the arc of $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

50. A semi circular plate with diameter 4 feet is submerged vertically in the water with its diameter 5 feet below the surface of the water. Find the force exerted by the water on one side of the plate.

51. The region bounded by the graphs of $y = 2\sqrt{x}$, $y = 0$, and $x = 3$ is revolved about the *x*-axis. Find the surface area of the solid generated.

52. A leaky 5-lb bucket is lifted from the ground into the air by a worker pulling in 20 ft of rope at a constant speed. The rope weighs 0.7 lb/ft. The bucket starts with 2 gallons of water (16 lb) and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was done in lifting the bucket to the top?

53. A parabolic mirror is modeled by revolving the curve $y = 2\sqrt{x}$ (where x and y are measured in feet), $0 \le x \le c$, about the x-axis. If it needs to have 50 square feet of surface area, determine c.

54. A vertical drain plate is at the bottom of one end wall of a rectangular swimming pool whose dimensions are $50\times30\times10$ ft. The plate is an inverted isosceles triangle whose base is 2 feet and whose height is 1 foot. Water is running into the pool at a rate of 1000 ft^3/hr. The plate is designed to withstand a force of 520 pounds. To what depth can the pool be filled without exceeding this design limitation?

55. A region is bounded by $y = e^x$, the x-axis, and the lines $x = 0$ and $x = 1$. Find the perimeter of the region.

56. A worker weighing 180 lb climbs the scaffolding on a bridge that is being painted. Attached to his waist is a safety cable that is anchored to a point on the bridge that is 60 feet over his head. The cable, which weighs 0.8 lb/ft is fully extended when he begins his climb. How much work is done in climbing the scaffolding to a height of 40 feet.

REVIEW: INTEGRATION TECHNIQUES

57. $\int \frac{x^2+1}{x^2+4x+3}dx =$

58. $\int_0^4 x\sqrt{2x+1}\,dx =$

59. $\int \frac{x^3}{\sqrt{x^2+1}}dx =$

60. $\int \frac{x}{(x+4)^2}dx =$

61. $\int (9-x^2)^{\frac{3}{2}}\,dx =$

62. $\int \frac{\ln(2x)}{x^2}dx =$

63. $\int \frac{x^2dx}{x^3-5x^2+2x-10} =$

64. $\int \cos x \ln(\sin x)dx =$